

Argonne National Laboratory

ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK

Numerical Determination of the
Space, Time, Angle, or Energy Distribution
of Particles in an Assembly

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BENCHMARK PROBLEM BOOK

Numerical Determination of the
Space, Time, Angle, or Energy Distribution
of Particles in an Assembly

Prepared by the
Benchmark Problem Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

February 1968

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PREFACE

This book is an outgrowth of activities of the Benchmark Problem Committee of the Mathematics and Computation Division of the American Nuclear Society. After much discussion and consideration of sample benchmark problems, the committee decided to restrict this first publication to a specific area which encompasses a major portion of reactor-physics computations.

The objectives and their implementation are described in the introductory material. A few sample benchmark problems prepared by committee members have been included in the initial book skeleton.

The structure and mechanism for maintaining this book were designed to facilitate use of the benchmark problems in a variety of ways without excessive paperwork. This first effort is an attempt to establish a workable framework upon which this and other benchmark problem books may be developed.

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the best way to do this is to start by identifying your strengths and interests. You can then focus on how you can use these to help you succeed in your chosen field. This will not only make you more successful, but also help you to feel more confident in your abilities.

Another key factor in achieving success is to have a positive attitude. This can be achieved by focusing on your strengths and interests, and by being open to new opportunities and challenges. By doing this, you will be more likely to succeed and achieve your goals.

Finally, it's important to remember that success is not always guaranteed. It requires hard work, dedication, and persistence. However, if you stay focused on your goals and continue to work hard, you will eventually achieve success.

Conclusion
In conclusion, success is not something that just happens. It requires hard work, dedication, and persistence. By identifying your strengths and interests, having a positive attitude, and staying focused on your goals, you can increase your chances of success. Remember, success is not guaranteed, but it is definitely achievable with the right mindset and effort.

ARGONNE CODE CENTER:
BENCHMARK PROBLEM BOOK

Numerical Determination of the
Space, Time, Angle, or Energy Distribution
of Particles in an Assembly

I. OBJECTIVES

This initial collection contains benchmark problems relating to numerical determination of space, time, angle, or energy distribution of particles in an assembly.

The primary objective is to provide reliable solutions to diverse problems of varying complexity. This book should thus serve as a source-book of solutions to mathematically well-defined problems for which either analytical or very accurate approximate solutions are known.

Secondary objectives are:

1. To assist in evaluation of specific computer programs.
2. To facilitate comparison of specific computers.
3. To provide standard test problems for use in verifying transmitted programs.
4. To aid in the assessment of sensitivity to, and accuracy of, physical parameters.
5. To provide target problems useful in the development of numerical-solution techniques or methods of error analysis.

II. MECHANISM

The Argonne Code Center (ACC) is the central benchmark-problem distributing agency, and all inquiries should therefore be directed to the ACC. Proposed problems and additional data on accepted problems will be forwarded by the ACC to the Benchmark Problem Committee (BPC). The ACC will distribute problems and solutions to organizations on request. An attempt will be made to update this book periodically.

No problem will be accepted by the BPC without a complete documented solution, and an independent solution may sometimes be required. The originator of a problem must review solutions for two years. He will deal directly with alternative solutions and will transmit them to the BPC

in appropriate form. Solutions transmitted to the problem originator in proper form must be relayed to the BPC. Such solutions may be rejected only by the BPC. The reviewer is expected to obtain evidence supporting the validity of a reported solution.

After every two-year period, a reviewer may continue in this capacity for another two years, appoint a successor subject to the approval of the BPC, or request that the BPC appoint another reviewer.

III. GUIDELINES AND FORMAT

A. Source Situations

Certain problems are given a special status by the BPC and are designated as source situations. A source situation is a concise description of a particular configuration from which a number of benchmark problems may be derived. These source situations are created in an attempt to interrelate benchmark problems, thus providing some continuity in the benchmark book. The source situations should simplify preparation of derived problems and enhance meaningful comparisons.

A problem originator should, if feasible, derive his problem from an existing source situation. The BPC may reject a problem not derived in this manner if the committee decides that an appropriate and not too inconvenient source situation was available. Any problem with an identification number starting with a zero is a problem for which there is no source situation. New source situations will be drawn from such problems.

B. Format

Although there is no rigid form for benchmark problems, it is recommended that the following outline be used as a guide:

1. Benchmark Problem Identification

Number: (BPC fills in)

Acceptance Date: (BPC fills in)

Submitted by: _____ Date: _____

Descriptive Title: _____

2. Theory

Description

Equations

- Constraints
 - Boundary conditions
 - Approximations and simplifications
3. Configuration
- Description
 - Reduction from source situation, if applicable
 - Sketch
4. Data
5. Specific Problems of Interest
- Reference problem
 - Other problems
6. Expected Results
- Primary results
 - Auxiliary results
7. Summary of Available Solutions
- Most accurate known solution with the estimated accuracy
 - Other solutions
8. Documentation of Solutions
- References
 - Sufficient data on techniques to enable duplication of results
 - Computer characteristics, including essential hardware and software data for the program with which the problem was solved, and significant figures carried in the computation
 - Solution details
- Rigid adherence to any form might hinder rather than aid the specification of benchmark problems. For this reason, format guidelines rather than standard forms have been presented for general use.

IV. BENCHMARK PROBLEMS

Source Situations

1. Small Spherical Critical Experiment
2. A High-temperature Gas-cooled Reactor Configuration
3. An Analytical 2-D Multigroup Diffusion Problem
4. A Simple Highly Nonseparable Reactor

BENCHMARK SOURCE SITUATION

Identification: 1

(To be filled in by Benchmark Committee)

Date Submitted: July 1966

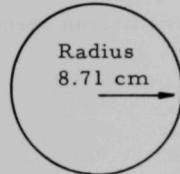
By: K. D. Lathrop (GGA)
(Name and Organization)

Date Adopted: August 1, 1966

By: D. R. Vondy (ORNL)
(Name and Organization)Descriptive Title: Small Spherical Critical Experiment (Lady Godiva)

Suggested Function: Test transport theory solutions and multigroup cross-section sets

Configuration



$$\begin{aligned}\psi(r, \mu) &= 0 \\ \text{at } r &= 8.71 \text{ cm} \\ \text{for } \mu &< 0\end{aligned}$$

Details

Homogeneous atomic densities per cm³:U²³⁵: 0.045447×10^{24} U²³⁸: 0.00256×10^{24}

Reference

1. R. E. Peterson and G. A. Newby, An Unreflected 235-U Critical Assembly, Nucl. Sci. Eng. 1(2), 112 (May 1956).

BENCHMARK PROBLEMS

Identification: 1-Al

Source Situation ID.1

Date Submitted: July 1966

By: K. D. Lathrop (GGA)
(Name and Organization)

Date Accepted: August 1, 1966

By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Multigroup Transport Theory

Reduction of Source Problem

1. Multigroup approximation made
 2. Isotropic scattering assumed

Data: Hansen-Roach¹ six-group cross sections and fission spectrum

Fission Spectrum	Velocities	Group
.2040000E-00	.2850000E+02	1
.3440000E-00	.1990000E+02	2
.1680000E-00	.1470000E+02	3
.1800000E-00	.1100000E+02	4
.9000000E-01	.6700000E+01	5
.1400000E-01	.2900000E+01	6

Additional Data

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
<u>U²³⁵ Cross Sections</u>					
σ_a	1.26000E+00	1.30000E+00	1.33000E+00	1.35000E+00	1.66000E+00
$\nu\sigma_f$	3.55700E+00	3.19600E+00	3.08700E+00	2.98800E+00	3.51800E+00
σ_t	4.25000E+00	4.50000E+00	4.65000E+00	5.20000E+00	7.90000E+00
$\sigma_{g \rightarrow g}$	1.20000E+00	1.77000E+00	2.30000E+00	3.42000E+00	6.16000E+00
$\sigma_{g-1 \rightarrow g}$.00000E 00	2.70000E-01	2.40000E-01	5.50000E-01	3.50000E-01
$\sigma_{g-2 \rightarrow g}$.00000E 00	.00000E 00	3.70000E-01	6.70000E-01	4.00000E-01
$\sigma_{g-3 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	6.50000E-01	4.50000E-01
$\sigma_{g-4 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	.00000E 00	4.40000E-01
$\sigma_{g-5 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	.00000E 00	6.00000E-02
<u>U²³⁸ Cross Sections</u>					
σ_a	5.66000E-01	5.35000E-01	1.44000E-01	1.40000E-01	1.60000E-01
$\nu\sigma_f$	1.72500E+00	1.21300E+00	1.08000E-01	.00000E 00	.00000E 00
σ_t	4.00000E+00	4.40000E+00	4.50000E+00	5.25000E+00	8.20000E+00
$\sigma_{g \rightarrow g}$	1.25400E+00	1.82500E+00	2.90600E+00	4.53000E+00	7.96000E+00
$\sigma_{g-1 \rightarrow g}$.00000E 00	3.30000E-01	3.50000E-01	8.00000E-01	5.00000E-01
$\sigma_{g-2 \rightarrow g}$.00000E 00	.00000E 00	4.60000E-01	9.60000E-01	5.50000E-01
$\sigma_{g-3 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	7.90000E-01	6.40000E-01
$\sigma_{g-4 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	.00000E 00	5.30000E-01
$\sigma_{g-5 \rightarrow g}$.00000E 00	.00000E 00	.00000E 00	.00000E 00	7.00000E-02

Expected Primary Results

1. Multiplication factor
2. Apparent asymptotic convergence rate
3. Number of iterations, total and outer
4. Machine time (total and iteration only)
5. Ratio of surface leakage to total losses by energy group

Possible Additional Results

1. Dependence of results on space and/or angular quadrature scheme
2. Dependence of execution time on numerical refinement of solution
3. Scalar and angular fluxes

Best Solution Available: Converged integral equation solution

Reference

1. G. E. Hansen and W. H. Roach, Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies, Los Alamos Scientific Laboratory Report LAMS-2543 (Dec 1961).

Solutions

1. Discrete Ordinates: 1-A1-1, 1-A1-2, 1-A1-3, and 1-A1-4
2. Monte Carlo: 1-A1-5

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-1

Benchmark Problem ID.1-A1

Date Submitted: July 1966

By: K. D. Lathrop (GGA)
(Name and Organization)

Date Accepted: August 1, 1966

By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Multigroup Discrete Ordinates Calculation

Mathematical Model: Discrete ordinates,¹ diamond difference scheme,
multigroup solution of transport equation

Pertinent Features of Techniques Used

Gauss Legendre quadrature,² uniform space mesh. Group rebalancing. Relative change in pointwise scalar flux required to be less than 10^{-6} over entire system for convergence.

Computer: IBM-7030

Date Solved: July 1966

at: Los Alamos
(Installation)

Program: DTF-IV

References

1. K. D. Lathrop, DTF-IV, A Fortran-IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering, Los Alamos Scientific Laboratory Report LA-3373 (Nov 1965).

2. M. Abramowitz and I. A. Stegun, Eds., Handbook of Mathematical Functions, NBS, Appl. Math. Sci. 55, USGPO, Washington, pp. 916-919 (1964).

Primary Results (Space intervals 40, angular quadrature order 16)

1. Multiplication factor 0.996679

2. Apparent asymptotic convergence rate (reciprocal of the number of iterations required to reduce apparent absolute error by a factor of e) 0.80 based on outer iterations

3. Number of iterations, 761 total, 17 outer, where total counts each group individually

4. Machine time, total 3.00 min, iteration only (~2.8) min
 5. Ratio of surface leakage to total losses by energy group

<u>Group</u>	<u>Fraction Surface Leakage</u>
1	0.079975885
2	0.14685806
3	0.091228941
4	0.14725990
5	0.091788806
6	<u>0.011237521</u>
Total	0.56834912

Additional Results

EXHIBIT A: Multiplication factor vs space and angular quadrature

EXHIBIT B: Execution time vs space and angular quadrature

EXHIBIT C: DTF-IV listing for 40 intervals and order 16 quadrature (available from ACC on request)

EXHIBIT A

Number of Space Intervals, I	Order of Angular Quadrature, N						
	4	8	12	16	24	32	48
10	1.00506	0.998564	0.997223	0.996722	0.996347	0.996212	0.996113
20	1.00514	0.998564	0.997204	0.996695	0.996317	0.996179	0.996079
40	1.00514	0.998554	0.997189	0.996679	0.996298	0.996161	0.996060
80	1.00515	0.998550	0.997185	0.996674	0.996293	0.996155	0.996055
160	1.00515	0.998549	0.997184	0.996673	0.996292	0.996154	0.996054

Extrapolated multiplication factor 0.99604.

EXHIBIT B - (IBM-7030)

Total machine execution time (min, to nearest sec) as dependent on
 order of angular quadrature and number of geometric space intervals.^a

Number of Space Intervals, I	Order of Angular Quadrature, N						
	4	8	12	16	24	32	48
10	0.333	0.517	0.700	0.883	0.233	1.583	2.300
20	0.583	0.900	1.483	1.583	2.233	2.900	4.217
40	1.050	1.700	2.317	3.000	4.367	5.500	7.883 (0.567)
80	2.017	3.267	4.483	5.717	8.033	10.367	15.067 (1.166)
160	3.883	6.333	8.567	10.950	15.550	20.367	29.733 (2.433)

^aMachine time includes that for edit of angular fluxes. (This time is shown in parentheses for a few cases.)

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-2

Benchmark Problem ID.1-A1

Date Submitted: August 1966

By: H. Greenspan (ANL)
(Name and Organization)

Date Accepted: August 15, 1966

By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Multigroup Discrete Ordinates Calculation

Mathematical Model: See Ref. 1

Pertinent Features of Techniques Used

See Ref. 4 for angular quadrature. Change in total fission source between outer iterations required to be less or equal to 10^{-6} for convergence.

Computer: CDC-3600

Date Solved: October 1966

at: ANL
(Installation)

Program: SNARG-1D, DTF-IV

References

1. G. J. Duffy, H. Greenspan, S. D. Sparck, J. V. Zapatka, and M. K. Butler, SNARG-1D, A One-dimensional Discrete-ordinate, Transport-theory Program for the CDC-3600, ANL-7221 (June 1966).
2. B. G. Carlson, W. J. Worlton, W. Guber, and M. Shapiro, DTF Users Manual, UNC Phys.-Math-3321 (1963).
3. B. G. Carlson, Solution of the Transport Equation of S_n Approximations, LA-1891, Los Alamos Scientific Laboratory (1955).
4. C. E. Lee, The Discrete S_n Approximation to Transport Theory, LA-2595 (March 1962).
5. K. D. Lathrop, DTF-IV, A Fortran-IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering, Los Alamos Scientific Laboratory Report LA-3373 (Nov 1965).
6. K. D. Lathrop and B. G. Carlson, Discrete Ordinates Angular Quadrature of the Neutron Transport Equation, LA-3186, Los Alamos Scientific Laboratory (1965).

Primary Results (Space intervals 40, angular quadrature order 16)

1. Multiplication factor 0.997020

2. Apparent asymptotic convergence rate (reciprocal of the number of iterations required to reduce apparent absolute error by a factor of e) 0.80 based on outer iterations

3. Number of iterations, 569 total, 17 outer, where total counts each group individually

4. Machine time, total 2.15 min, iteration only 1.85 min

Additional Results

EXHIBIT A: Multiplication factor as a function of angular quadrature order N, and number of space intervals I, using SNARG-1D

EXHIBIT B: DTF-IV and SNARG-1D results for k using different angular quadratures as a function of angular quadrature order N, for 40 space intervals with the CDC-3600 computer

EXHIBIT C: Iteration time (sec) and total time (sec) vs angular quadrature order N and number of space intervals I

EXHIBIT D: Fluxes available from ACC upon request

EXHIBIT A

Multiplication factor as a function of angular quadrature order N, and number of space intervals I, using SNARG-1D.^a

Number of Space Intervals, I	Order of Angular Quadrature, N				
	4	8	12	16	24
10	1.00615	0.999034	0.997635	0.997066	0.996595
20	1.00622	0.999031	0.997613	0.997038	0.996562
40	1.00623	0.999019	0.997597	0.997020	0.996543
80	1.00623	0.999015	0.997592	0.997016	0.996538
160	1.00623	0.999015	0.997591	0.997015	

Extrapolated multiplication factor 0.99630.

^aSNARG standard angular quadrature.⁴

EXHIBIT B

DTF-IV and SNARG-1D results for k , using different angular quadratures as a function of angular quadrature order N , for 40 space intervals with the CDC-3600 computer.

Code	Order of Angular Quadrature, N					
	8			16		
	Standard ^a	Lathrop ^b	Gauss ^c	Standard	Lathrop	Gauss
DTF-IV	0.999019	0.999300	0.998554	0.997020	0.997210	0.996679
SNARG-1D	0.999019	0.999300	0.998553	0.997020	0.997210	0.996679

^aAngular quadrature from Ref. 4.

^bAngular quadrature from Ref. 6, Table 1.

^cM. Abramowitz and J. A. Stegun, eds., *Handbook of Mathematical Functions*, NBS Applied Mathematics Series 55, pp. 916-919.

EXHIBIT C

Iteration time and total time vs angular quadrature order N and number of space intervals I.

Number of Space Intervals, I	Order of Angular Quadrature, N														
	4			8			12			16			24		
	Total No. of Iterations	Iteration Time, sec	Total Time, sec	Total No. of Iterations	Iteration Time, sec	Total Time, sec	Total No. of Iterations	Iteration Time, sec	Total Time, sec	Total No. of Iterations	Iteration Time, sec	Total Time, sec	Total No. of Iterations	Iteration Time, sec	Total Time, sec
10	558(17)	13	30	549(16)	18	36	549(16)	25	47	549(16)	31	53	549(16)	42	61
20	572(17)	23	41	565(17)	35	57	565(17)	46	68	565(17)	57	75	565(17)	80	99
40	575(17)	43	65	569(17)	66	88	569(17)	87	110	569(17)	111	129	569(17)	154	174
80	575(17)	87	106	286(12) ^a	84	107	162(9) ^a	67	91	8316 ^a	57	80	570(17)	306	442
160	575(17)	167	188	286(12) ^a	161	197	162(9) ^a	137	166	8316 ^a	112	156			

Number in parentheses indicates number of outer iterations.

^aStarting values from preceding problem were used.

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-3

Benchmark Problem ID.1-A1

Date Submitted: October 1966

By: W. W. Engle (UCCTC)
(Name and Organization)

Date Accepted: October 1966

By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Multigroup Discrete Ordinates Calculation

Mathematical Model: Full-range Gauss Legendre quadrature

Pertinent Features of Techniques Used

See Ref. 1. Maximum relative change in scalar flux between inner iterations and of source between outer iteration of 10^{-6} .

Computer: IBM-7090 and
IBM-360/75Date Solved: October 1966
at: UCCTC
(Installation)

Program: ANISN

Other References

1. W. W. Engle, M. A. Boling, and B. W. Colston, DTF-II, A One-Dimensional, Multigroup Neutron Transport Program, NAA-SR-1051.

2. B. G. Carlson, W. J. Worlton, W. Guber, and M. Shapiro, DTF Users Manual, UNC Phys. Math-3321 (1963).

3. W. W. Engle, A Users Manual for ANISN, Union Carbide Computer Technology Center, Report K-1693, March 30, 1967.

Primary Results

Computer	IBM-7090	IBM-360/75
(Space intervals 40, angular quadrature order 16)		
1. Multiplication factor	0.996674	0.996666
2. Number of iterations,		
Outer	17	17
Total	412	467

Computer	IBM-7090	IBM-360/75
(Space intervals 40, angular quadrature order 16)		
3. Machine time (min)		
Total	~5	1.32
Iteration only	~4	0.93
4. Apparent asymptotic convergence rate (reciprocal of the number of iterations to reduce the absolute error by a factor of e)	0.80	0.80
5. Ratio of surface leakage to total losses		

Group	IBM-7090	IBM-360/75
1	0.0799757	0.0799753
2	0.146858	0.146856
3	0.0912286	0.0912276
4	0.147259	0.147256
5	0.091788	0.0917863
6	0.0112374	0.0112371
Total	0.568346	0.568339

Additional Results

Space Intervals	Quadrature Order	IBM-7090	IBM-360/75
(Multiplication factor)			
10	4	1.00506	1.00505
20	8	0.998534	0.998534
40	16	0.996674	0.996666
80	32	0.996152	0.996133
160	64		0.995968
Extrapolated infinitesimal		0.99612	0.99596
(Machine time, min)			
Total (iteration only)			
10	4	0.39(0.10)	
20	8	0.58(0.26)	
40	16	1.32(0.93)	
80	32	4.32(3.76)	
160	64	13.62(12.66)	

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-4

Benchmark Problem ID.1-A1

Date Submitted: January 1967

By: K. D. Lathrop (LASL)
(Name and Organization)

Date Accepted: May 1967

By: K. D. Lathrop (GGA)
(Name and Organization)

Descriptive Title: Multigroup Spherical Integral Equation Solution

Mathematical Model: Solution of differenced version of integral equation

Pertinent Features of Techniques Used

Direct outer iteration for scalar flux only. No inner iteration.
 Relative convergence on pointwise scalar flux of 10^{-6} .

Computer: IBM-7030

Date Solved: January 1967

at: LASL
(Installation)

Program: Special program called MGSPIN

Primary Results

<u>Number of Intervals</u>	<u>Multiplication Factor</u>
20	0.9955227
40	0.9958613
80	0.9959467
160	0.9959661
320	0.9959702

Additional Results

Problem listing for 80 space intervals available from ACC on request.

BENCHMARK PROBLEM SOLUTION

Identification: 1-A1-5

Benchmark Problem ID.1-A1

Date Submitted: July 1966

By: G. E. Whitesides (UCCTC)
(Name and Organization)

Date Accepted: October 12, 1966

By: D. R. Vondy (ORNL)
(Name and Organization)Descriptive Title: Discrete Energy, Monte Carlo Reactivity Calculation for
a Metal Uranium Sphere

Mathematical Model: Monte Carlo

Computer: IBM-360/75 - 360/50

Date Solved: July 1966

at: Union Carbide Computer
Technology Center
(Installation)

Program (With references)

Unpublished code KENO, by G. E. Whitesides, Union Carbide.

Summary of Available Solutions

Benchmark		Computer	Code	Machine Time, min	Neutron Histories Followed	Multiplication Factor
Problem No.	Solution					
1-A1-5A	IBM-360/50	KENO		10	32,565	.99724 ± .00690
1-A1-5B	IBM-360/75	KENO		10	187,875	.99112 ± .00286
1-A1-5C	IBM-360/75	KENO		19	467,100	.99458 ± .00193
1-A1-5D	IBM-360/75	KENO		120	3,290,000	.99525 ± .00072

Results

CASE	5A	5B	5C	5D
Neutron Histories Followed	32,565	187,875	467,100	3,290,000
Calculated Multiplication Factor	0.99724	0.991116	0.994576	0.995246
Statistical Error Bound (1 Standard Deviation)	0.00690	0.00286	0.00193	0.00072
Surface Leakage Group				
1	0.078102	0.080570	0.080009	0.079898
2	0.14799	0.14798	0.14672	0.14686
3	0.089853	0.090335	0.091038	0.09125
4	0.14564	0.14842	0.14740	0.14771
5	0.094199	0.092197	0.092545	0.091483
6	0.010560	0.010742	0.011102	0.011301
Total	0.56635	0.57025	0.56882	0.56851
Absorption Group				
1	0.050351	0.049825	0.049341	0.049843
2	0.096839	0.096216	0.096611	0.096384
3	0.062037	0.061383	0.060525	0.060585
4	0.10283	0.10215	0.10338	0.10389
5	0.094746	0.094268	0.095655	0.095242
6	0.026850	0.025911	0.025670	0.025550
Total	0.43365	0.42975	0.43118	0.43149
Computer	360/50	360/75	360/75	360/75
Machine Time, min	10	10	19	120

BENCHMARK SOURCE SITUATION

Identification: 2
(To be filled in by Benchmark Committee)

Date Submitted: August 1966 By: Reimar Froehlich (GGA)
(Name and Organization)

Date Adopted: November 1967 By: Eugene Wachspress (KAPL)
(Name and Organization)

Descriptive Title: High-temperature Gas-cooled Reactor Problem

Suggested Function: Designed to test capabilities of multigroup programs

Configuration

Three-dimensional configuration including space dimensions and region numbers (two figures)

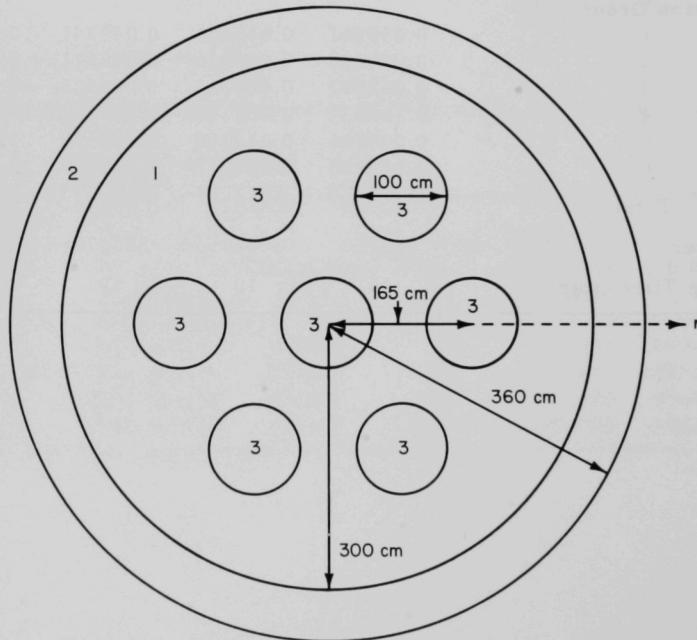
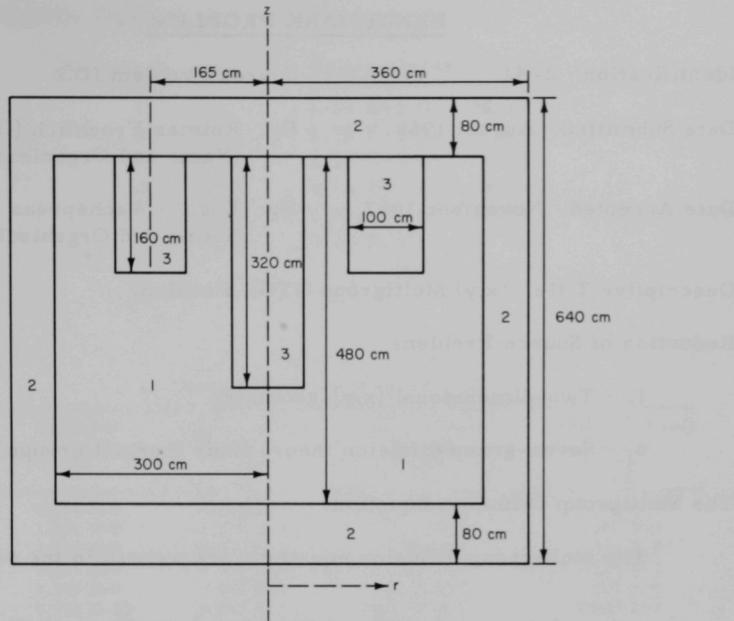


Fig. 1. (r, θ) Picture

Fig. 2. (r, z) Picture ($\theta = 0, \pi/3, 2\pi/3$)

Boundary Conditions: Vacuum boundary condition on outside surface

Temperatures for the Regions

Region	Temperature, °K
1	1100
2	900
3	1100

Isotope Concentrations for the Different Regions

Notation of Concentrations in 10^{24} atoms/cm³

Isotope	Region 1	Region 2	Region 3
Thorium-232	3.91 E-4	0	3.91 E-4
Uranium-235	2.22 E-5	0	2.22 E-5
Uranium-238	1.54 E-6	0	1.54 E-6
Boron (natural)	0	0	2.22 E-4
Boron-10	3.76 E-7	0	3.76 E-7
Carbon	6.03 E-2	8.03 E-2	6.03 E-2

BENCHMARK PROBLEM

Identification: 2-A1

Source Problem ID.2

Date Submitted: August 1966

By: Reimar Froehlich (GGA)
(Name and Organization)

Date Accepted: November 1967

By: Eugene Wachspress (KAPL)
(Name and Organization)

Descriptive Title: (x,y) Multigroup HTGR Problem

Reduction of Source Problem

1. Two-dimensional (x,y)-geometry
2. Seven-group diffusion theory (four thermal groups)

The Multigroup Diffusion Equations

The multigroup diffusion equations are written in the form

$$-\nabla \cdot D^g \nabla \phi^g + \Sigma_t^g \phi^g - \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_s^{g' \rightarrow g} \phi^g \phi^{g'} = \frac{1}{\lambda} \chi^g \sum_{g'=1}^G \nu \Sigma_f^{g' \rightarrow g} \phi^{g'},$$

with

$$\Sigma_t^g = \Sigma_c^g + \Sigma_f^g + \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_s^{g' \rightarrow g}.$$

Additional Data

Fission Spectrum

$$\chi_1 = 0.9675, \chi_2 = 0.0325, \chi_3 = \chi_4 = \chi_5 = \chi_6 = \chi_7 = 0.$$

Boundary Conditions

$$D^g \frac{\partial \phi^g}{\partial n} = -\delta^g \phi^g$$

(n is the outward directed normal on the surface)

 D^g is the diffusion coefficient of the adjacent region

$$\delta^g = 0.4695 \text{ for } g = 1, 2, \dots, 7.$$

The Energy Groups

Energy Group	Energy Interval, eV
1	1.83 E+5 to 1.49 E+7
2	1.76 E+1 to 1.83 E+5
3	2.38 E+0 to 1.76 E+1
4	4.14 E-1 to 2.38 E+0
5	1.00 E-1 to 4.14 E-1
6	4.00 E-2 to 1.00 E-1
7	0 to 4.00 E-2

Region 1
Macroscopic Cross Sections

Group, g	Diffusion Coefficient, D^g	Capture Cross Section, Σ_c^g	Fission Cross Section, Σ_f^g	Production Cross Section, $\nu \Sigma_f^g$
1	2.384 E+0	5.328 E-5	4.286 E-5	1.074 E-4
2	1.221 E+0	1.726 E-3	2.553 E-4	6.207 E-4
3	1.227 E+0	9.050 E-4	7.504 E-4	1.823 E-3
4	1.225 E+0	8.905 E-4	1.277 E-3	3.104 E-3
5	1.145 E+0	2.453 E-3	4.127 E-3	1.003 E-2
6	9.930 E-1	4.023 E-3	7.115 E-3	1.729 E-2
7	9.175 E-1	7.479 E-3	1.395 E-2	3.390 E-2

From Group Into Group	Scattering Matrix $\Sigma_s^{g' \rightarrow g}$						
	1	2	3	4	5	6	7
1	1.198 E-2						
2		3.546 E-3					
3			2.140 E-2				
4				3.643 E-2	1.821 E-4	1.890 E-4	
5					1.049 E-2	1.856 E-2	2.305 E-3
6						1.234 E-4	5.946 E-2
7							4.051 E-5
							2.257 E-2
							4.869 E-2

Remark: $\Sigma_s^{g' \rightarrow g}$ is zero if the position in the table is blank.

Region 2
Macroscopic Cross Sections

Group, g	Diffusion Coefficient, D_g	Capture Cross Section, Σ_c^g	Fission Cross Section, Σ_f^g	Production Cross Section, $\nu \Sigma_f^g$
1	1.839 E+0	5.621 E-8	0.0	0.0
2	9.480 E-1	2.497 E-6	0.0	0.0
3	9.329 E-1	2.028 E-5	0.0	0.0
4	9.432 E-1	6.043 E-5	0.0	0.0
5	8.932 E-1	1.172 E-4	0.0	0.0
6	7.971 E-1	1.910 E-4	0.0	0.0
7	7.730 E-1	3.419 E-4	0.0	0.0

From Group Into Group	Scattering Matrix $\Sigma_s^{g' \rightarrow g}$						
	1	2	3	4	5	6	7
1	1.562 E-2						
2		6.369 E-3					
3			2.986 E-2				
4				4.936 E-4	7.600 E-2	3.616 E-4	3.757 E-5
5					7.031 E-3	2.893 E-2	3.421 E-3
6					7.066 E-5	6.481 E-2	1.899 E-2
7					2.137 E-5	2.209 E-2	5.513 E-2

Remark: $\Sigma_s^{g' \rightarrow g}$ is zero if the position in the table is blank.

Region 3
Macroscopic Cross Sections

Group, g	Diffusion Coefficient, D_g	Capture Cross Section, Σ_c^g	Fission Cross Section, Σ_f^g	Production Cross Section, $\nu \Sigma_f^g$
1	2.381 E+0	6.942 E-5	4.286 E-5	1.074 E-4
2	1.217 E+0	2.184 E-3	2.553 E-4	6.207 E-4
3	1.222 E+0	1.916 E-3	7.504 E-4	1.823 E-3
4	1.221 E+0	1.800 E-3	1.277 E-3	3.104 E-3
5	1.142 E+0	3.193 E-3	4.127 E-3	1.003 E-2
6	9.909 E-1	4.737 E-3	7.115 E-3	1.729 E-2
7	9.158 E-1	8.224 E-3	1.395 E-2	3.390 E-2

From Group Into Group	Scattering Matrix $\Sigma_s^{g' \rightarrow g}$						
	1	2	3	4	5	6	7
1	1.200 E-2						
2		3.551 E-3					
3			2.140 E-2				
4				7.827 E-4	3.643 E-2	1.821 E-4	1.890 E-4
5					1.049 E-2	1.856 E-2	2.305 E-3
6					1.234 E-4	5.946 E-2	1.490 E-2
7					4.051 E-5	2.257 E-2	4.869 E-2

Remark: $\Sigma_s^{g' \rightarrow g}$ is zero if position in table is blank.

Expected Primary Results

1. Maximum eigenvalue (k -effective)
2. Fundamental flux distribution
3. Computation time and convergence rates if applicable

Possible Additional Results

1. Neutron balance
2. Higher mode calculations
3. Averaged region fluxes

Best Solution Available: The "maximum sigma-total variance scheme," described in the solution form 2-A1-1 below guarantees a trustworthy solution

Solutions

1. Benchmark problem solution 2-A1-1.

BENCHMARK PROBLEM SOLUTION

Identification: 2-A1-1

Benchmark Problem ID.2-A1

Date Submitted: June 1967

By: Reimar Froehlich (GGA)
(Name and Organization)

Date Accepted: November 1967

By: Eugene Wachspress (KAPL)
(Name and Organization)

Descriptive Title: Discrete (x,y) Multigroup HTGR Problem

Mathematical Model

1. The usual five-point difference equations (box integration method)
2. See the mesh specifications in Figs. 3 and 4 for a quarter of the assembly

Pertinent Features of Techniques Used

1. Successive line overrelaxation method with automatic adjustment of overrelaxation factor
2. One type of iterations only
3. Improvement of the eigenvalue after each iteration using the overall neutron balance equation
4. Special flux extrapolation

Computer: UNIVAC 1108

Date Solved: February 1967

at: General Atomic
(Installation)

Program: GAMBLE-5

References

1. J. P. Dorsey and R. Froehlich, GAMBLE-5, A Program for the Solution of the Multigroup Neutron-Diffusion Equations in Two Dimensions, with Arbitrary Group Scattering, for the UNIVAC 1108 Computer, GA-8188 (1967).

2. J. P. Dorsey, GAMBLE-4, A Program for the Solution of the Multigroup Neutron-Diffusion Equations in Two Dimensions, with Arbitrary Group Scattering, GA-6540.

Primary Results

	<u>Coarse Mesh</u>		<u>Fine Mesh</u>	
Number of space meshpoints		1,369		5,429
Number of energy-space meshpoints		9,583		39,003
Absolute largest eigenvalue	1.006397	1.006398	1.006419	1.006418
Convergence criteria				
Maximum relative change in group fluxes for successive iterations	0.0001	0.000011	0.000074	0.000019
Relative change of eigenvalue for successive iterations	0.0000005	0.0000003	0.0000006	0.0000003
Total number of (inner) iterations	72	105	127	181
Computer time in minutes				
Total	6.45	8.45	24.45	34.45
Iterations only	5.0	7.0	22.3	32.3

Additional Results

EXHIBIT A: Maximum $\delta\Sigma_t$

EXHIBIT B: Averaged region fluxes

EXHIBIT A

Maximum $\delta\Sigma_t$

For iterative solution methods for which there is not a complete theoretical understanding of the convergence behavior, an iteration-independent error measure for the achieved convergence is very desirable. For example, the residuals of the difference equations can be used. Another possibility is the maximum sigma-total variation, which will be defined below.

Let $\bar{\Sigma}_t(p)$ be the volume-weighted total removal cross section for the energy-space meshpoint p . If an eigenvalue λ and flux values for each

energy-space meshpoint p are given, then it is easily possible to calculate a p -dependent quantity $\tilde{\Sigma}_t(p)$ that fulfills the difference equations exactly for each energy-space meshpoint p . In other words, the given eigenvalue and flux values (approximate solutions of the original problem) are the exact solutions of a modified problem with $\bar{\Sigma}_t(p)$ replaced by $\tilde{\Sigma}_t(p)$ (which could be accomplished by a p -dependent sigma-absorption variation). If the modified problem is essentially not different from a practical reactor-design standpoint, the reactor designer might be satisfied having the solution of that modified problem.

The maximum $\delta\Sigma_t$ is defined as

$$\text{Maximum } \delta\Sigma_t = \underset{\text{all } p}{\text{Max}} \left| \frac{\tilde{\Sigma}_t(p) - \bar{\Sigma}_t(p)}{\bar{\Sigma}_t(p)} \right|.$$

This is a measure for the necessary sigma-total variation, that is, how different the two problems are.

Problem	Coarse Mesh		Fine Mesh	
Number of iterations	72	105	127	181
Maximum $\delta\Sigma_t$	0.00027	0.000033	0.00033	0.00017

EXHIBIT B

Averaged Region Fluxes

1. Coarse-mesh Problem (105 Iterations)

Group No.	Region No.		
	1	2	3
1	8.527	1.056	7.535
2	18.357	3.405	16.408
3	2.913	0.732	2.543
4	2.907	1.002	2.424
5	5.335	7.215	4.092
6	1.416	3.066	1.043
7	0.379	1.032	0.273

2. Fine-mesh Problem (181 Iterations)

1	8.6048	0.9328	7.6753
2	18.4850	3.1899	16.7050
3	2.9280	0.6966	2.5876
4	2.9357	0.9921	2.4655
5	5.3363	7.4586	4.1608
6	1.4063	3.1975	1.0609
7	0.3746	1.0808	0.2778

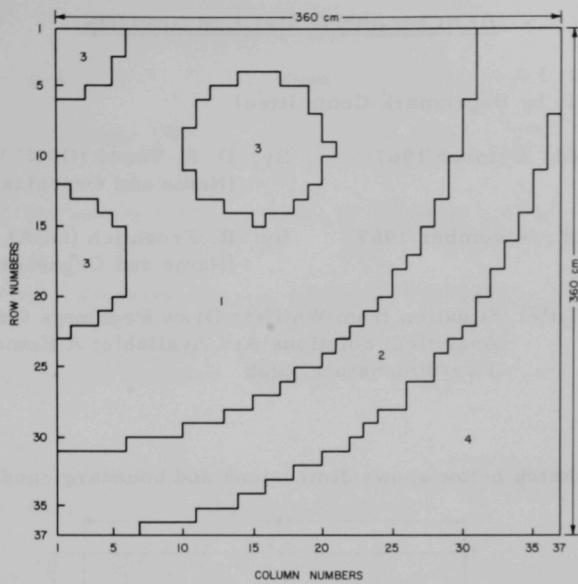


Fig. 3. Coarse Mesh for Benchmark Problem

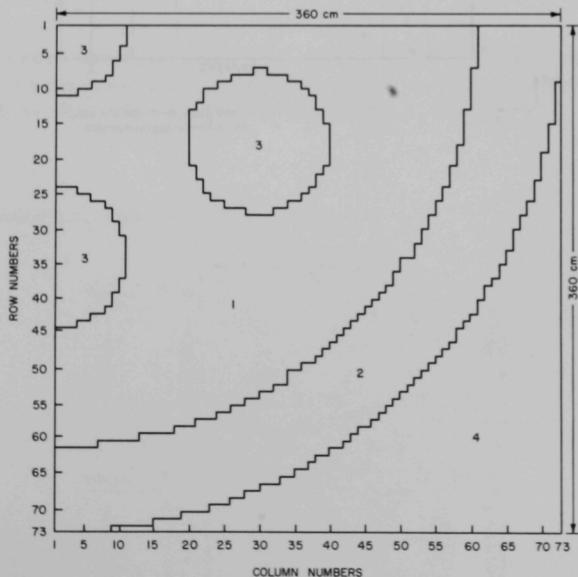


Fig. 4. Fine Mesh for Benchmark Problem

BENCHMARK SOURCE SITUATION

Identification: 3

(To be filled in by Benchmark Committee)

Date Submitted: October 1967

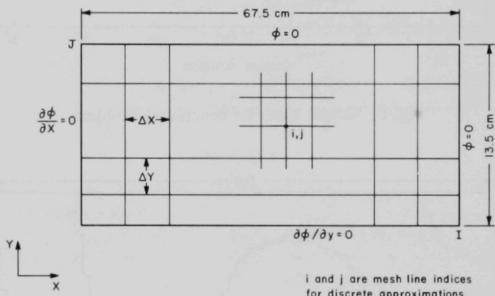
By: D. R. Vondy (ORNL)
(Name and Organization)

Date Accepted: November 1967

By: R. Froehlich (GGA)
(Name and Organization)Descriptive Title: Situation from Which to Draw Problems for Which
Analytical Solutions Are Available: A Homogeneous,
Two-dimensional Slab

Configuration

The sketch below shows dimensions and boundary conditions.



BENCHMARK PROBLEM

Identification: 3-A1

Source Situation ID.3

Date Submitted: October 1967

By: D. R. Vondy (ORNL)
(Name and Organization)

Date Adopted: November 1967

By: R. Froehlich (GGA)
(Name and Organization)

Descriptive Title: Discrete Form of a Simple Multigroup, Two-dimensional Neutron Diffusion Problem

Reduction from Source Situation: None

Theory

The equations for multigroup two-dimensional neutron-diffusion eigenvalue problems for a Homogeneous Slab are

$$-\frac{D_g \partial^2 \phi_{i,g}}{\partial x^2} - \frac{D_g \partial^2 \phi_{i,g}}{\partial y^2} + \left[h \Sigma_{a,g} + \sum_n \Sigma(g \rightarrow n) \right] \phi_{i,g} = \sum_n \Sigma(n \rightarrow g) \phi_{i,n}$$

$$+ \frac{\chi_g}{k_e} \sum_n \nu \Sigma_{f,n} \phi_{i,n},$$

where usual cross sections have been used, i refers to a geometric location and n and g to discrete energy groups, k_e is the effective multiplication factor, and h is the relative absorption cross section.

Data

Macroscopic data are given below.

Group Macroscopic Cross Sections ^a					
Group, n	D _n	$\Sigma_{a,n}$	$\Sigma_{r,n}$	$\nu \Sigma_{f,n}$	χ_n
1	1.6	0.01	0.04	0.02	0.90
2	1.4	0.02	0.06	0.01	0.09
3	1.2	0.03	0.06	0.02	0.01
4	1.0	0.10	0.06	0.04	0
5	0.8	0.05	0.08	0.10	0
6	0.6	0.07	0.10	0.12	0
7	0.4	0.09	0.08	0.15	0

Transfer Matrix $\Sigma(n \rightarrow g)$							
Group, n	g						
	1	2	3	4	5	6	7
1	0	0.015	0.01	0.01	0.005	0	0
2	0	0	0.02	0.02	0.01	0.01	0
3	0	0	0	0.02	0.02	0.01	0.01
4	0	0	0	0	0.03	0.02	0.01
5	0	0	0	0	0	0.05	0.03
6	0	0	0	0	0.02	0	0.08
7	0	0	0	0	0.02	0.06	0

^a $\Sigma_{r,n} = \sum_g \Sigma(n \rightarrow g).$

Specific Problems of Interest

Determine k_e and point flux values for several finite-difference mesh structures for h unity.

Determine h and point flux values for several finite-difference mesh structures for k_e unity given h unity at the start.

Expected Results

1. Dependence of the eigenvalues on mesh structure and on the number of iterations for flux values initialized at unity
2. Computation time
3. Validity of convergence estimators

Analytical Solution Available: Yes

Remark: Conventional methods of reducing the three thermal groups to two (or one) may be studied with this problem and results obtained with programs that will not treat the full up-scattering problem.

Summary of Available Solutions

Benchmark Problem Solution No.	Computer	Computer Code	Mesh I x J	k_e	Solutions		
					Machine Time, min	h	Machine Time, min
3-A1-1		(Precise)	Continuum	0.7745451357	-	0.7376078371	-
3-A1-2			9 x 3	0.8049537413	-	0.7758858250	-
3-A1-3			24 x 6	0.7779379571	-	0.7419289881	-
3-A1-4			69 x 15	0.7749222647	-	0.7380887903	-
3-A1-5			204 x 42	0.7745870407	-	0.7376612865	-
3-A1-A2	IBM-360/75	EXTERMINATOR-2	9 x 3	0.804955	0.13	0.775887	0.15
3-A1-A3			24 x 6	0.777937	1.26	0.741928	1.30
3-A1-A4			69 x 15	0.774934	14.5	0.738053	15.6
3-A1-A5			204 x 42	0.776809	274.0	(0.73349)	293.0
3-A1-A4DP		(Special) ^a	69 x 15	0.774920	17.3	0.738081	15.6
3-A1-B3	IBM-360/75	CITATION ^b	(22 x 4)	0.77884	0.12		
3-A1-B4			(67 x 13)	0.77503	2.69		
3-A1-B5			(202 x 40)	0.77361	77.4		
3-A1-C1	UNIVAC 1108	GAMBLE 5 ^c	71 x 17	0.775491	2.8 ^d		
3-A1-C2			71 x 17	0.7755010	3.5		
3-A1-C3			191 x 43	0.7752492	12.2		

^aEXTERMINATOR-2 all in double precision, equivalent 17 decimal digits.

^bMeshpoints are at center of discrete volumes, precise results are not applicable, and these results are preliminary.

^cFirst mesh point at boundary and 0.01 cm extrapolation distance used at 0 flux boundary, precise results not applicable.

^dThis problem was treated by row relaxation, and problems 3-A1-C2 and -C3 were treated by column relaxation.

BENCHMARK PROBLEM SOLUTION

Identification: 3-A1-1, 3-A1-2, Benchmark Problem ID.3-A1
 3-A1-3, 3-A1-4,
 3-A1-5

Date Submitted: October 1967 By: D. R. Vondy (ORNL)
 (Name and Organization)

Date Accepted: November 1967 By: R. Froehlich (GGA)
 (Name and Organization)

Descriptive Title: Solutions of Benchmark Problem 3-A1, Analytical
 Formulation

The usual five-point central-difference formulation was used with leakage estimated for finite elements of volume from first derivatives approximated midway between points, and this same formulation was used at boundaries (leakage at edge is thereby estimated from slope between two outermost points). Equal mesh spacing was used along each direction. Zero-derivative boundaries were located halfway between the first two mesh points (between numbers 1 and 2) so that the first and last points along each direction do not involve independent unknowns.

Analytical Equations

The finite-difference equations can be solved if it is recognized that a cosine source yields a cosine flux.¹⁻³ As formulated, the eigenvalues are given by the determinate associated with the set of N equations for N groups,

$$(D_n B^2 + h \sum_{a,n} + \sum_{r,n}) X_n = \frac{\chi_n}{k_e} \sum_g \nu \sum_{f,g} X_g + \sum_g \sum_{(g \rightarrow n)} X_g,$$

where X is a spacially independent variable and g and n refer to discrete groups and for the continuum (Solution 3-A1-1):

$$B^2 = \left(\frac{\pi}{2H}\right)^2 + \left(\frac{\pi}{2W}\right)^2,$$

and for the finite-difference mesh with uniform spacing,

$$B^2 = \left[\frac{2J-3}{H} \sin\left(\frac{\pi/2}{2J-3}\right)\right]^2 + \left[\frac{2I-3}{W} \sin\left(\frac{\pi/2}{2I-3}\right)\right]^2,$$

where H is the half-height (13.5) and W is the half-width (67.5).

To normalize the flux values to a total neutron production rate of unity, the required factors are given by

$$G_n = X_n \div \left(AWH \sum_n \nu \Sigma f, n X_n \right),$$

where for the continuum $A = 4/\pi^2$, and for the finite-difference mesh with uniform spacing,

$$A = \left[(2I - 3)(2J - 3) \tan\left(\frac{\pi/2}{2I - 3}\right) \tan\left(\frac{\pi/2}{2J - 3}\right) \right]^{-1}.$$

Point flux values are given by

$$\phi_{i,j,n} = G_n \cos\left[\frac{\pi}{2}\left(\frac{2i - 3}{2I - 3}\right)\right] \cos\left[\frac{\pi}{2}\left(\frac{2j - 3}{2J - 3}\right)\right].$$

These equations apply¹ only if simple weighting of flux values is used; that is, a point flux applies over its associated finite-difference volume, and the mesh is as shown in the problem description. Higher-order flux weighting would be expected to give improved finite-difference results.

It has been pointed out by Froehlich⁴ that if the first mesh point in each direction lies at the actual boundary, the above equations modify as follows (solutions were not obtained for these):

$$\text{The finite mesh: } B^2 = \left[\frac{2(J-1)}{H} \sin\left(\frac{\pi/4}{J-1}\right) \right]^2 + \left[\frac{2(I-1)}{W} \sin\left(\frac{\pi/4}{I-1}\right) \right]^2,$$

$$\Phi_{i,j,g} = G_g \cos\left[\frac{\pi}{2}\left(\frac{i-1}{I-1}\right)\right] \cos\left[\frac{\pi}{2}\left(\frac{j-1}{J-1}\right)\right], \text{ and}$$

$$A = \left[4(I-1)(J-1) \tan\left(\frac{\pi/4}{I-1}\right) \tan\left(\frac{\pi/4}{J-1}\right) \right]^{-1}.$$

Primary Results

Solution No.	Mesh (I x J)	$k_e(h = 1)$	$h(k_e = 1)$
3-A1-1	Continuum	0.7745451357	0.7376078371
3-A1-2	9 x 3	0.8049537413	0.7758858250
3-A1-3	24 x 6	0.7779379571	0.7419289881
3-A1-4	69 x 15	0.7749222647	0.7380887903
3-A1-5	204 x 42	0.7745870407	0.7376612865

Other Results

Values of G_n and selected point-flux values are given below for problems; normalization was to one-neutron production over the quadrant considered with account of the finite mesh. Point-flux values with more significant figures at any location could readily be made available; they are easily calculated from the data and equations presented.

Group-dependent Normalization Factors and Flux Values at Selected Points
(All values $\times 10^5$)

Location (i,j)	Energy Group						
	1	2	3	4	5	6	7
ID.3-A1-1	Continuum (at mesh points for the 69×15 mesh)						
$h = 1$							
G_g	4338.0184	968.11811	619.65226	431.61507	501.60025	385.93727	321.32897
$\phi_g(55,10)$	762.95710	170.26959	108.98250	75.911108	88.219883	67.877440	56.514335
$\phi_g(5,12)$	1478.7711	330.01821	211.23097	147.13167	170.9866	131.56073	109.53665
$k_e = 1$							
G_g	3486.1115	811.53489	543.29591	419.08390	514.68488	447.89619	400.58771
$\phi_g(55,10)$	613.12639	142.73022	95.553185	73.707165	90.521166	78.774580	70.454114
$\phi_g(5,12)$	1177.2711	276.64113	185.20214	142.85996	175.44903	152.68168	136.55487
ID.3-A1-2	Mesh (9×3)						
$h = 1$							
G_g	4744.6112	1067.6724	687.66102	477.50044	557.78495	430.50291	358.68642
$\phi_g(7,2)$	1671.2621	376.08148	242.22465	168.19679	196.47655	151.64218	126.34523
$\phi_g(2,2)$	4086.4445	919.56619	592.26952	411.26216	480.40971	370.78407	308.92988
$k_e = 1$							
G_g	3994.3746	902.67214	616.31077	466.41602	570.64424	488.65845	432.55657
$\phi_g(7,2)$	1389.3833	324.30148	217.09194	164.29237	201.00616	172.12714	152.36557
$\phi_g(2,2)$	3397.2157	792.95763	530.81690	401.71536	491.48517	420.87235	372.55286
ID.3-A1-3	Mesh (24×6)						
$h = 1$							
G_g	4378.5147	978.07542	626.74348	436.21268	507.24018	390.41562	325.08390
$\phi_g(19,5)$	512.18893	114.41309	73.283475	51.027190	59.335830	45.669952	38.027593
$\phi_g(5,4)$	2730.8537	610.01984	390.72779	272.06326	316.36269	243.49991	202.75290
$k_e = 1$							
G_g	3532.2805	822.54292	550.66810	423.84757	520.30594	451.95697	403.74514
$\phi_g(19,5)$	413.19832	96.219243	64.415931	49.580747	60.864233	52.868922	47.229210
$\phi_g(5,4)$	2203.0624	513.01514	343.44843	264.35122	324.51173	281.88288	251.81345
ID.3-A1-4	Mesh (69×15)						
$h = 1$							
G_g	4342.4665	969.21234	620.40209	432.12041	502.22028	386.42966	321.74184
$\phi_g(55,10)$	763.73942	170.46203	109.11438	75.999986	88.328933	67.964041	56.586948
$\phi_g(5,12)$	1480.2874	330.01821	211.23097	147.13167	170.98866	131.56073	109.53665
$k_e = 1$							
G_g	3491.1892	812.74571	544.10690	419.60776	515.30289	448.34224	400.93419
$\phi_g(55,10)$	614.01943	142.94317	95.695820	73.799300	90.629861	78.853030	70.515052
$\phi_g(5,12)$	1190.0986	277.05388	185.47859	1430.3854	175.65970	152.83373	136.67298

(Contd. next page)

Group-dependent Normalization Factors and Flux Values at Selected Points (Contd.)
 (All values $\times 10^5$)

Location (i,j)	Energy Group						
	1	2	3	4	5	6	7
ID.3-A1-5	Mesh (204 x 42)						
$h = 1$							
G_g	4338.5120	968.23955	619.73547	431.67115	501.66906	385.99192	321.37479
$\phi_g(163,10)$	1283.6082	286.46693	183.35723	127.71582	148.42566	114.20099	95.083133
$\phi_g(5,34)$	1324.1723	295.51976	189.15162	131.75185	153.11616	117.80994	98.087918
$k_e = 1$							
G_g	3486.6750	811.66928	543.38592	419.14204	514.75347	447.94569	400.62616
$\phi_g(163,10)$	1031.5805	240.14347	160.76816	152.29686	132.53086	118.53073	106.22593
$\phi_g(5,34)$	1064.1802	247.73240	165.84871	127.92780	157.10970	136.71906	122.27650

Numerical Solution Method

Results were obtained by iterative solution of the system of equations with an IBM-360/75 computer carrying 17 decimal digits equivalent significance.

References

1. M. L. Tobias, The Exact Solution to the Finite-Difference Approximation to the Multigroup Diffusion Equations for a One-Region Slab Reactor, ORNL-TM-1729, Oak Ridge National Laboratory (Dec 1966).
2. E. L. Wachspress, Iterative Solution of Elliptic Systems, Prentice-Hall Inc., New Jersey (1966).
3. S. P. Frankel, Convergence Rates of Iterative Treatments of Partial Differential Equations, Math. Tables Aids Compt., 4:65 (1950).
4. R. Froehlich, private communication.

BENCHMARK PROBLEM SOLUTION

Identification: 3-A1-A2, 3-A1-A3, Benchmark Problem ID.3-A1
 3-A1-A4, 3-A1-A5,
 3-A1-A4DP

Date Submitted: November 1967 By: T. B. Fowler (ORNL)
 (Name and Organization)

Date Accepted: November 1967 By: D. R. Vondy (ORNL)
 (Name and Organization)

Descriptive Title: Iterative Solutions for Multigroup Two-dimensional
 Neutron-diffusion Problems

Finite-difference Approximations

Five-point, central-difference formulation, identical with that of
 the analytical solutions 3-A1-A2, -A3, -A4, -A5.

Iterative Technique

Direct iteration of the nonlinear equations with successive over-relaxation, using the overall neutron balance for the iterate estimates of the eigenvalue of the problem; line relaxation (forward-backward substitution) used along direction of maximum mesh points.

Special Acceleration Techniques

Successive overrelaxation with partitioning by a line at an energy; point-flux extrapolation assuming a single error mode.

Initialization

Unknown flux values set equal and the overrelaxation coefficient initialized at

$$\beta_0 = \frac{2}{1 + \sin \frac{\pi}{J-1}},$$

where J is the total number of mesh points normal to the direction of line relaxation. For the direct criticality search, h was initialized at unity and no other information supplied.

Program Name: EXTERMINATOR-2 (ORNL)

Type of Program: General-purpose, two-space-dimension, multigroup

Program Language: FORTRAN IV

Machine Language Contents: None

Program Ancestors: EXTERMINATOR

Computer: IBM-360/75

Hardware Used: 128K-4 byte-fast core, 1024-bulk, one-tape; two disks for largest problem

Computer System: IBM-OS360

Compiler: IBM-FORTRAN IV, H-Level 22, 512 K version (July 1966)

Significant Figures Carried: Equivalent seven-decimal

Convergence Criteria: Maximum relative flux change 5×10^{-6} (not always satisfied)

Reference

1. T. B. Fowler, M. L. Tobias, and D. R. Vondy,
EXTERMINATOR-2: A FORTRAN-IV Code for Solving Multigroup Neutron Diffusion Equations in Two Dimensions, ORNL-4078 (April 1967).

Primary Results: Dependence of eigenvalues on iteration count (machine time is proportional to iteration count)

Usual Eigenvalue Problems				Criticality Search Problems		
Iterations	k_e	Maximum Relative Flux Change	Apparent Absolute Convergence ^a	h	Maximum Relative Flux Change	Apparent Absolute Convergence ^a
<u>ID.3-A1-A2</u>				Mesh (9 x 3)		
1	0.969644	0.993	-	0.995774	0.993	-
20	0.803261	1.30-02	1.07-01	0.773745	1.29-02	9.58-02
40	0.804927	2.32-04	2.38-03	0.775885	1.67-05	1.78-04
56	0.804955	4.35-06	2.42-05			
60				0.775887	6.20-06	7.78-05
70				0.775887	4.47-06	3.24-05
Machine time, min:				0.15		

^aEstimate of the maximum relative error in any point flux from the iterative behavior (single error mode).

Primary Results (Contd.):

Iterations	ke	Usual Eigenvalue Problems			Criticality Search Problems		
		Maximum Relative Flux Change	Apparent Absolute Convergence ^a	h	Maximum Relative Flux Change	Apparent Absolute Convergence ^a	
<u>ID.3-A1-A3</u>		Mesh (24 x 6)					
1	0.769069	0.938	-	0.960469	0.938	-	
20	0.773794	2.13-02	2.01-01	0.737309	2.10-02	1.84-01	
40	0.777613	1.59-03	3.45-02	0.742019	1.55-03	4.95-03	
60	0.777954	8.49-05	1.98-03	0.741904	1.17-04	1.76-03	
80	0.777942	2.29-05	1.60-04	0.741923	3.44-05	7.97-04	
100	0.777939	7.69-06	5.08-05	0.741928	1.16-05	8.98-05	
120	0.777936	6.91-06	6.86-05	0.741931	1.05-05	1.26-04	
142	0.777937	4.17-06	1.95-05	0.741930	4.53-06	2.60-05	
Machine time, min: 1.26				1.30			
<u>ID.3-A1-A4</u>		Mesh (69 x 15)					
1	0.6113301	0.676	-	0.919945	0.676	-	
20	0.762374	3.66-02	0.195	0.720227	3.43-02	2.24-01	
40	0.768247	1.19-02	0.239	0.732591	7.60-03	1.79	
60	0.772806	2.58-03	1.94	0.734425	3.46-03	0.109	
80	0.773330	1.63-03	0.144	0.735409	2.35-03	0.152	
100	0.773734	1.30-03	9.39-02	0.738018	2.46-04	3.08-03	
120	0.774942	3.85-05	1.51-04	0.738015	5.71-05	2.88-05	
160	0.774938	1.60-05	2.90-04	0.738041	3.31-05	1.45-03	
183	0.774935	9.95-06	7.64-05				
200	0.774934	8.40-06	1.89-04	0.738053	1.86-05	3.04-04	
Machine time, min: 14.5				15.6			
<u>ID.3-A1-A5</u>		Mesh ^b (204 x 42)					
1	0.374448	0.716	-	0.79089	0.716	-	
20	0.71938	9.69-02	0.666	0.71654	9.28-02	0.610	
40	0.76928	0.294	1.19	0.72502	0.257	0.901	
60	0.76092	8.37-03	1.68-02	0.72085	8.95-03	4.39-02	
80	0.76231	4.94-03	4.60-01	0.72211	4.68-03	3.44-02	
100	0.76392	3.44-03	4.11-01	0.72983	5.24-03	1.83-03	
120	0.76520	2.85-03	1.43-01	0.73003	1.27-03	8.67-03	
160	0.76710	2.13-03	3.51-01	0.73201	1.09-03	6.16-04	
200	0.76809	1.26-03	1.54-01	0.73349	1.85-03	2.65-02	
Machine time, min: 274				293			

^bNote that these problems are not converged to a usually acceptable degree.

Additional Results

Point-flux values agreed to three significant figures when maximum relative flux change was below 10^{-5} .

Remarks

These problems exhibit very slow rates of convergence compared with those usually encountered.

Special Calculations

The whole code was modified to calculate in double precision, equivalent to 17 decimal digits. Results obtained for the 15×69 mesh are shown in the table below.

Iterations	k _e	Usual Eigenvalue Problem			Criticality Search Problem		
		Maximum Flux Change	Apparent Absolute Convergence	Absolute Relative Error in k _e	h	Maximum Flux Change	Apparent Absolute Convergence
ID.3-A1-A4DP							
1	0.613332	0.676		0.21	0.919954	0.676	0.25
20	0.762380	3.66-02	0.195	0.016	0.720229	3.43-02	0.224
40	0.768255	1.19-02	0.238	0.0086	0.732580	7.55-02	2.15
60	0.772801	2.59-03	1.16	0.0027	0.734423	3.46-03	0.106
80	0.773327	1.63-03	0.141	0.0021	0.738011	3.35-04	1.59-03
100	0.775249	2.85-04	0.185	0.00042	0.737967	9.95-05	7.68-05
120	0.775194	2.13-04	4.24-02	0.00035	0.738045	8.62-05	2.48-03
154	0.774900	1.66-05	1.02-03	0.00029	0.738081	4.80-06	1.20-04
185	0.774920	2.07-06	3.10-05	0.000003			0.000011
Machine time, min: 17.3						15.6	

BENCHMARK PROBLEM SOLUTION

Identification: 3-A1-B3, 3-A1-B4, Benchmark Problem ID.3-A1
3-A1-B5

Date Submitted: November 1967 By: T. B. Fowler (ORNL)
(Name and Organization)

Date Accepted: November 1967 By: D. R. Vondy (ORNL)
(Name and Organization)

Descriptive Title: Iterative Solutions for Multigroup Two-dimensional
Neutron Diffusion Problems

Finite-difference Approximations: Five-point, central-difference formulation
carried to boundaries, meshpoints at centers of discrete volumes, uniform
spacing of all points, no points located
on boundaries

Iterative Technique

Direct iteration of the nonlinear equations with successive over-relaxation using the overall neutron balance for the iterate estimates of the eigenvalue of the problem; line relaxation (forward-backward substitution) used along direction of maximum meshpoints.

Special Acceleration Techniques

Successive overrelaxation with partitioning by a line at an energy; point flux extrapolation assuming a single error mode.

Initialization

Unknown flux values set equal, the eigenvalue initialized at unity, and the overrelaxation coefficient initialized at

$$\beta_0 = \frac{2}{1 + \sin \frac{\pi}{J+1}},$$

where J is the total number of internal meshpoints normal to the direction of line relaxation.

Program Name: CITATION (ORNL)

Type of Program: General-purpose reactor-core depletion code

Program Language: FORTRAN IV

Machine Language Contents: None

Computer: IBM-360/75

Hardware Used: 128K-4 byte-fast core, 1024K-bulk storage used for the largest problem

Computer System: IBM-OS360

Compiler: IBM-FORTRAN IV, H-Level 22, 512 K version (July 1966).

Significant Figures Carried: Equivalent seven-decimal

Convergence Criteria: Maximum relative flux change $< 10^{-4}$

Reference

1. T. B. Fowler and D. R. Vondy, Nuclear Reactor Depletion and Kinetics Code CITATION, report to be issued from ORNL.

Primary Results: Dependence of eigenvalues on iteration count (machine time is proportional to iteration count)

Usual Eigenvalue Problems			
Iterations	k_e	Maximum Relative Flux Change	Apparent Absolute Convergence ^a
<u>ID.3-A1-B3</u>			Mesh (22 x 4) ^b
1	0.71570	4.4	
20	0.77618	1.6-02	0.16
40	0.77870	8.9-04	1.3-02
45	0.77884	5.9-05	4.7-04
Machine time, min: 0.12			
<u>ID.3-A1-B4</u>			Mesh (67 x 13)
1	0.61924	1.7	
40	0.7694	1.0-02	0.40
80	0.7739	1.1-03	2.0-02
100	0.77424	8.6-04	7.7-02
116	0.77503	9.4-05	8.7-04
Machine time, min: 2.69			

^aEstimate of maximum absolute error.

^bThese are internal points.

Primary Results (Contd.):

Usual Eigenvalue Problems			
Iterations	k_e	Maximum Relative Flux Change	Apparent Absolute Convergence ^a
<u>ID.3-A1-B5</u> Mesh (202 x 40)			
1	0.36088	0.95	
40	0.75778	1.8	1.1
80	0.75926	4.2-03	0.16
120	0.76222	2.7-03	0.18
160	0.77015	1.1-03	6.3-04
200	0.77026	4.3-04	2.7-02
300	0.77155	3.2-04	7.5-02
400	0.77361	7.6-05	1.2-02
Machine time, min: 77.4			

BENCHMARK PROBLEM SOLUTION

Identification: 3-A1-C1, 3-A1-C2, Benchmark Problem ID.3-A1
3-A1-C3

Date Submitted: November 1967 By: R. Froehlich (GGA)
(Name and Organization)

Date Accepted: December 1967 By: H. Greenspan (ANL)
(Name and Organization)

Descriptive Title: Iterative Solutions for Multigroup Two-dimensional
Neutron-diffusion Problems

Finite-difference Approximation

The usual five-point difference equations were used with the first mesh point along each direction located on the zero-derivative boundary and equal spacing of points in each direction. A 0.01 cm extrapolation distance was used at zero-flux boundaries.

Solution Technique: A line overrelaxation technique using the overall neutron balance for the iterate estimates of the eigenvalue k_{eff}

Special Acceleration Techniques

1. Automatic adjustment of the overrelaxation factor
2. Periodic application of rebalancing techniques for group-columns and group-rows. The rebalancing is used alternately for group-columns or group-rows.
3. Special asymptotic flux extrapolation assuming a single error mode

Initialization: Flux values are set equal to one; overrelaxation factor β is set equal to 1.6

Program Name: GAMBLE-5

Type of Program: General purpose, two-space-dimension, multigroup code

Program Language: FORTRAN IV

Machine Language Contents: The program includes some assembly language subroutines for efficient data handling

Program Ancestors: GAMBLE-4, GAMBLE-1, EXTERMINATOR

Computer: UNIVAC 1108

Hardware Used: 65,356 words of core storage, at least three magnetic tape units (including the program tape) on one data channel, 1,572,864 words of FH-880 drum storage from one data channel

Operating System: General Atomic version of (GA X 23) of the 1108 Exec II operating system

Significant Figures Carried: Equivalent of eight decimals

Convergence Criteria: Maximum relative flux change less than 10^{-5}

Reference

1. J. P. Dorsey and R. Froehlich, GAMBLE-5, A Program for the Solution of the Multigroup Neutron-diffusion Equations in Two Dimensions, with Arbitrary Group Scattering, for the UNIVAC 1108 Computer, GA-8188 (1967).

Primary Results: Dependence of eigenvalues on iteration count (machine time is proportional to iteration count)

Iterations	k _{eff}	Maximum Relative Flux Change	Absorption Variance	Machine Time, min
		Iterative Total		
<u>ID.3-A1-C1^a</u>				
		Mesh (71 x 17)		
Two Rebalancings				
1	0.763741	0.096422	-	
5	0.769807	0.010900	-	
10	0.774339	0.005022		
One Rebalancing				
11	0.775489	0.000037	-	
14	0.775495	0.000007	0.000059	2.8 3.8
<u>ID.3-A1-C2</u>				
		Mesh (71 x 17)		
Two Rebalancings				
1	0.725119	0.484726	-	
5	0.743801	0.087871	-	
10	0.753582	0.042201	-	
One Rebalancing				
11	0.775500	0.000041	-	
18	0.775501	0.000007	0.000015	3.5 5.1
<u>ID.3-A1-C3</u>				
		Mesh (191 x 43)		
Two Rebalancings				
1	0.706300	0.252950	-	
5	0.714809	0.047821	-	
10	0.722907	0.025608	-	
One Rebalancing				
11	0.775248	0.000049	-	
17	0.775249	0.000006	0.000022	12.2 15.5

^aThis problem was treated by row relaxation, and problems 3-A1-C2 and -C3 were treated by column relaxation.

The eigenvalues are larger by about 0.0006 than the precise eigenvalues of comparable finite meshes. This is due to the nonzero extrapolation distance (≈ 0.01 cm); GAMBLE-5 does not have provision for exact zero boundary conditions.

BENCHMARK SOURCE SITUATION

Identification: 4
 (To be filled in by Benchmark Committee)

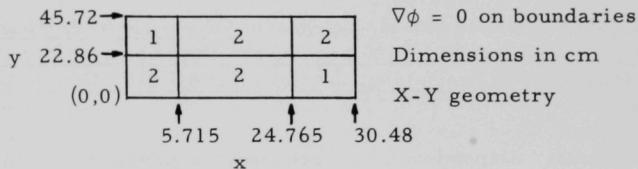
Date Submitted: January 16, 1967 By: Eugene Wachspress (KAPL)/
 F. C. Merriman (KAPL)
 (Name and Organization)

Date Adopted: November 1967 By: H. Greenspan (ANL)
 (Name and Organization)

Descriptive Title: Highly Nonseparable Reactor

Suggested Function: Designed to tax capabilities of synthesis approximations

Configuration



Details

Materials	Atoms/cc $\times 10^{-24}$						
	Zr	U^{235}	U^{238}	H	O	Zircaloy	Th
Region 1 is highly reactive:	.97261-02	.89719-03	.66676-04	.24937-01	.33727-01	.61176-02	-
Region 2 is much less reactive than Region 1:	-	-	-	.27452-01	.25927-01	.60801-02	.61003-02

BENCHMARK PROBLEM

Identification: 4-A1

Source Problem ID.4

Date Submitted: January 16, 1967

By: Eugene Wachspress (KAPL)/
F. C. Merriman (KAPL)
(Name and Organization)

Date Accepted: November 1967

By: H. Greenspan (ANL)
(Name and Organization)

Descriptive Title: Diffusion Theory for a Highly Nonseparable Reactor

Reduction of Source Problem

1. Diffusion theory: $-\nabla \cdot D \nabla \phi + A\phi = (\nu F/k)\phi$
2. Four-group macroscopic cross sections computed by MUFT-SWAK^{2,3}, in the KAPL-KARE system,¹ 5/7/65
3. Specific finite-difference network

Data

Grid:

27	1	2	2
14	2	2	1
1			

j 1 11 51 61 i

Five-point difference equations⁵Mesh-point indices: ij {
 $i = 1, 2, \dots, 61$
 $j = 1, 2, \dots, 27$
}Mesh increments: $\Delta x = 10$ at 0.5715,
 40 at 0.47625,
 10 at 0.5715
 $\Delta y = 26$ at 1.75846

Group Constants

REGION 1

$$\begin{array}{llll}
 D_1 = 2.07155 & A_1 = 0.0621200 & A_{2 \leftarrow 1} = 0.06067 & \nu F_1 = 0.00251 \\
 D_2 = 1.04356 & A_2 = 0.0585300 & A_{3 \leftarrow 2} = 0.05669 & \nu F_2 = 0.00328 \\
 D_3 = 0.82935 & A_3 = 0.0745100 & A_{4 \leftarrow 3} = 0.04268 & \nu F_3 = 0.04743 \\
 D_4 = 0.46506 & A_4 = 0.5628000 & & \nu F_4 = 1.1356
 \end{array}$$

 $\chi_1 = 0.7389580, \chi_2 = 0.2609630, \chi_3 = 0.0002212, \chi_4 = 0.0000000.$
 MUFT-11 Fission Spectrum (does not sum to unity).

Additional Data

REGION 2

$$\begin{array}{llll}
 D_1 = 2.28988 & A_1 = 0.0683 & A_{2 \leftarrow 1} = 0.0668 & \nu F_1 = 0.00172 \\
 D_2 = 1.22552 & A_2 = 0.06344 & A_{3 \leftarrow 2} = 0.06156 & \nu F_2 = 0.00 \\
 D_3 = 0.86183 & A_3 = 0.07185 & A_{4 \leftarrow 3} = 0.05985 & \nu F_3 = 0.00 \\
 D_4 = 0.603900 & A_4 = 0.0551 & & \nu F_4 = 0.00
 \end{array}$$

χ same as Region 1.

Four-group Matrix Showing Cross-section Subscript Significance

$$\left[\begin{array}{cccc}
 -\nabla \cdot D_1 \nabla + A_1 - \frac{\chi_1 \nu F_1}{k} & -\frac{\chi_1 \nu F_2}{k} & -\frac{\chi_1 \nu F_3}{k} & -\frac{\chi_1 \nu F_4}{k} \\
 -A_{2 \leftarrow 1} - \frac{\chi_2 \nu F_1}{k} & -\nabla \cdot D_2 \nabla + A_2 - \frac{\chi_2 \nu F_2}{k} & -\frac{\chi_2 \nu F_3}{k} & -\frac{\chi_2 \nu F_4}{k} \\
 -\frac{\chi_3 \nu F_1}{k} & -A_{3 \leftarrow 2} - \frac{\chi_3 \nu F_2}{k} & -\nabla \cdot D_3 \nabla + A_3 - \frac{\chi_3 \nu F_3}{k} & -\frac{\chi_3 \nu F_4}{k} \\
 -\frac{\chi_4 \nu F_1}{k} & -\frac{\chi_4 \nu F_2}{k} & -A_{4 \leftarrow 3} - \frac{\chi_4 \nu F_3}{k} & -\nabla \cdot D_4 \nabla + A_4 - \frac{\chi_4 \nu F_4}{k}
 \end{array} \right] \Phi = 0$$

A_g is the total removal for group g, $A_{g \leftarrow g'}$ is the slowing down from group g' into group g. The production matrix is

$$\begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{bmatrix} \left[\frac{\nu F_1}{k}, \frac{\nu F_2}{k}, \frac{\nu F_3}{k}, \frac{\nu F_4}{k} \right].$$

Expected Primary Results: Maximum eigenvalue (multiplication constant) - k. Fundamental flux vector - ϕ_{ijg} . Computation time and convergence rates when applicable.

Possible Additional Results: Higher mode computations

Best Solution Available: Converged 2-D computation ($k = 0.87914 \pm 0.00002$). Flux plots are in Ref. 4. Accuracy is adequate for synthesis comparisons.

References

- J. A. Archibald, Jr., and H. L. Teaford, KARE, A System of Diffusion Theory Programs for the Philco-2000, KAPL-2165-1 (1962).

2. H. Bohl, Jr., and A. P. Hemphill, MUFT-5-A Fast Neutron Spectrum Program for the Philco-2000, WAPD-TM-218 (1961).

3. F. D. Federighi and T. J. Reno, Jr., SWAK, A Thermal Cross-Section Program, KAPL-M-6145 (RFC-6, Rev. 1) (1964).

4. E. L. Wachspress, Some Mathematical Properties of the Multichannel Variational Synthesis Equations and Two-Dimensional Synthesis Numerical Studies, KAPL Memo 6588 (ELW-13) (1966).

5. E. L. Wachspress, Iterative Solution of Elliptic Systems, Prentice Hall (1966).

BENCHMARK PROBLEM SOLUTION

Identification: 4-A1-1

Benchmark Problem ID.4-A1

Date Submitted: February 1967

By: Eugene Wachspress (KAPL)/
F. C. Merriman (KAPL)
(Name and Organization)

Date Accepted: November 1967

By: H. Greenspan (ANL)
(Name and Organization)Descriptive Title: "Exact" Finite Difference Diffusion Calculation of a
Nonseparable Reactor

Mathematical Model: 2-D x-y Diffusion Calculation

Pertinent Features of Techniques Used: S.O.R. Inner Iteration; Chebyshev
Outer Iteration

Computer: CDC-6600

Date Solved: February 14, 1967

at: KAPL
(Installation)

Program (With references): NOVA

Other References: Ref. 4 of ID.4-A1

Primary Results

$$k = 0.87914 \pm 0.00002.$$

Six outer iterations with Chebyshev extrapolation based on an eigenvalue interval of $[0, \frac{1}{2}]$. For the last three iterations:

p	k_p	\sum all points and groups $\cdot \phi_p - \phi_{p-1} $
4	0.87898	855
5	0.87915	84.6
6	0.87914	2.13

35 inner iterations per group per outer iteration with S.O.R. extrapolation parameters: $\omega_1 = 1.5461$, $\omega_2 = 1.5005$, $\omega_3 = 1.40$, $\omega_4 = 1.40$.

Time per outer iteration: approximately 13 sec.

Total central processor time: approximately 3 min.

Additional Results

EXHIBIT A: Fluxes for thermal group (limited number available from ACC on request)

Group 4: Thermal Group--Topological Flux Edit

EXHIBIT A

J	I =	1	2	3	4	5	6	7	8	9	10	11	12
27	2.54666	2.54614	2.54636	2.55339	2.57994	2.65022	2.80941	3.14130	3.80120	5.07697	7.50257	9.51421	
26	2.53795	2.53743	2.53766	2.54469	2.57118	2.64129	2.80003	3.13096	3.78889	5.06083	7.47909	9.48471	
25	2.51182	2.51132	2.51158	2.51862	2.54495	2.61452	2.77194	3.09999	3.75205	5.01252	7.40880	9.39641	
24	2.46836	2.46788	2.46821	2.47524	2.50131	2.57000	2.72523	3.04850	3.69086	4.93231	7.29217	9.24992	
23	2.40766	2.40723	2.40764	2.41467	2.44040	2.50788	2.66008	2.97674	3.60563	4.82068	7.12998	9.04630	
22	2.32989	2.32951	2.33004	2.33709	2.36240	2.42836	2.57673	2.88502	3.49682	4.67835	6.92340	8.78712	
21	2.23525	2.23495	2.23563	2.24273	2.26756	2.33173	2.47555	2.77382	3.36511	4.50638	6.67417	8.47468	
20	2.12408	2.12387	2.12475	2.13194	2.15626	2.21843	2.35707	2.64385	3.21155	4.30637	6.38491	8.11249	
19	1.99703	1.99692	1.99807	2.00541	2.02926	2.08931	2.22232	2.49643	3.03798	4.08114	6.06018	7.70655	
18	1.85615	1.85619	1.85769	1.86530	1.88879	1.9777	2.07397	2.33480	2.84863	3.83669	5.70919	7.26862	
17	1.71059	1.71081	1.71278	1.72087	1.74426	1.80047	1.92216	2.16997	2.65621	3.58903	5.35415	6.82566	
16	1.60713	1.60762	1.61028	1.61923	1.64304	1.69811	1.81508	2.05091	2.51138	3.39287	5.05978	6.44910	
15	1.79850	1.79952	1.80370	1.81486	1.84089	1.89679	2.01039	2.23362	2.66384	3.48522	5.04923	6.34787	
14	3.67247	3.67532	3.68518	3.70625	3.74660	3.82002	3.94929	4.17154	4.54691	5.17238	6.20258	7.09268	
13	6.86390	6.86782	6.88053	6.90491	6.94586	7.01044	7.10775	7.24839	7.44275	7.69694	8.00389	8.27093	
12	7.94800	7.95048	7.95832	7.97271	7.99554	8.02920	8.07630	8.13919	8.21919	8.31570	8.42534	8.52186	
11	7.93255	7.93414	7.93907	7.94783	7.96116	7.97995	8.00516	8.03761	8.07780	8.12579	8.18114	8.23222	
10	7.41431	7.41613	7.42169	7.43126	7.44525	7.46420	7.48875	7.51953	7.55716	7.60224	7.65529	7.70595	
9	6.72384	6.72662	6.73501	6.74924	6.76964	6.79666	6.83088	6.87292	6.92350	6.98337	7.05336	7.12004	
8	6.03137	6.03537	6.04743	6.06773	6.09659	6.13443	6.18178	6.23927	6.30760	6.38757	6.48002	6.56729	
7	5.41576	5.42097	5.43666	5.46300	5.50028	5.54890	5.60935	5.68223	5.76823	5.86809	5.98264	6.08998	
6	4.90636	4.91263	4.93151	4.96316	5.00785	5.06594	5.13792	5.22433	5.32582	5.44310	5.57696	5.70179	
5	4.50752	4.51466	4.53614	4.57212	4.62283	4.68865	4.77000	4.86742	4.98153	5.11299	5.26254	5.40158	
4	4.21254	4.22034	4.24380	4.28306	4.33836	4.41005	4.49855	4.60436	4.72807	4.87033	5.03184	5.18170	
3	4.01136	4.01962	4.04446	4.08601	4.14451	4.22029	4.31377	4.42543	4.55584	4.70563	4.87549	5.03292	
2	3.89479	3.90332	3.92897	3.97187	4.03225	4.11044	4.20685	4.32195	4.45631	4.61053	4.78531	4.94719	
1	3.85665	3.86527	3.89119	3.93453	3.99554	4.07452	4.17189	4.28813	4.42379	4.57948	4.75588	4.91923	

J	I	=	13	14	15	16	17	18	19	20	21	22	23	24
27		11.12411	12.38694	13.35096	14.05873	14.54771	14.85084	14.99700	15.01150	14.91647	14.73122	14.47252	14.15496	
26		11.08996	12.34932	13.31090	14.01713	14.50532	14.80831	14.95490	14.97030	14.87657	14.69294	14.43615	14.12073	
25		10.98774	12.23675	13.19107	13.89269	14.37856	14.68119	14.82909	14.84721	14.75740	14.57869	14.32765	14.01865	
24		10.81822	12.05013	12.99247	13.68654	14.16867	14.47081	14.62100	14.64378	14.56061	14.39020	14.14885	13.85067	
23		10.58270	11.79097	12.71683	13.40060	13.87774	14.17945	14.33307	14.36259	14.28893	14.13036	13.90272	13.61996	
22		10.28310	11.46154	12.36673	13.03771	13.50887	13.81042	13.96885	14.00737	13.94628	13.80327	13.59372	13.33097	
21		9.92225	11.06512	11.94584	12.60194	13.06644	13.36837	13.53321	13.58325	13.53799	13.41441	13.22730	12.98949	
20		9.50443	10.60665	11.45967	12.09923	12.55677	12.85995	13.03303	13.09725	13.07119	12.97103	12.81085	12.60286	
19		9.03683	10.09430	10.91713	11.53904	11.98968	12.29517	12.47841	12.55946	12.55588	12.48295	12.35398	12.18050	
18		8.53319	9.54316	10.33418	10.93773	11.38158	11.69021	11.88508	11.98505	12.00656	11.96398	11.86976	11.73470	
17		8.02335	8.98445	9.74212	10.32581	10.76160	11.07243	11.27849	11.39744	11.44470	11.43367	11.37596	11.28159	
16		7.57967	8.48757	9.20523	9.76141	10.18141	10.48733	10.69840	10.83131	10.90047	10.91829	10.89546	10.84110	
15		7.38402	8.20461	8.84778	9.34480	9.72155	9.99959	10.19703	10.32909	10.40868	10.44670	10.45238	10.43355	
14		7.80673	8.37651	8.82743	9.18023	9.45212	9.65758	9.80887	9.91646	9.98931	10.03510	10.06038	10.07073	
13		8.52554	8.75594	8.95704	9.12792	9.27006	9.38634	9.48032	9.55584	9.61673	9.66663	9.70893	9.74661	
12		8.61913	8.71398	8.80428	8.88897	8.96789	9.04158	9.11106	9.17771	9.24303	9.30861	9.37598	9.44654	
11		8.28726	8.34580	8.40759	8.47260	8.54106	8.61338	8.69017	8.77214	8.86007	8.95474	9.05687	9.16710	
10		7.76277	7.82608	7.89623	7.97364	8.05880	8.15222	8.25444	8.36601	8.48747	8.61928	8.76184	8.91542	
9		7.19483	7.27824	7.37083	7.47313	7.58568	7.70903	7.84369	7.99014	8.14878	8.31995	8.50387	8.70063	
8		6.66440	6.77189	6.89032	7.02025	7.16220	7.31670	7.48420	7.66513	7.85980	8.06847	8.29123	8.52807	
7		6.20864	6.33917	6.48207	6.63789	6.80711	6.99020	7.18758	7.39961	7.62654	7.86856	8.12570	8.39782	
6		5.83919	5.98966	6.15369	6.33177	6.52436	6.73188	6.95471	7.19316	7.44743	7.71764	8.00377	8.30562	
5		5.55419	5.72084	5.90199	6.09810	6.30960	6.53687	6.78026	7.04003	7.31636	7.60931	7.91882	8.24464	
4		5.34590	5.52487	5.71907	5.92893	6.15484	6.39717	6.65624	6.93228	7.22543	7.53574	7.86309	8.20722	
3		5.20521	5.39280	5.59612	5.81559	6.05159	6.30446	6.57451	6.86194	7.16690	7.48938	7.82927	8.18626	
2		5.12426	5.31693	5.52563	5.75077	5.99273	6.25183	6.52837	6.82254	7.13448	7.46417	7.81148	8.17608	
1		5.09787	5.29222	5.50270	5.72972	5.97364	6.23480	6.51348	6.80989	7.12414	7.45621	7.80597	8.17310	

J	I =	25	26	27	28	29	30	31	32	33	34	35	36
27	13.79115	13.39196	12.96674	12.52349	12.06902	11.60911	11.14862	10.69161	10.24145	9.80090	9.37220	8.95713	
26	13.75923	13.36250	12.93985	12.49926	12.04751	11.59036	11.13265	10.67843	10.23104	9.79325	9.36727	8.95489	
25	13.66413	13.27482	12.85994	12.42736	11.98381	11.53498	11.08565	10.63982	10.20081	9.77133	9.35358	8.94930	
24	13.50789	13.13106	12.72922	12.31012	11.88036	11.44551	11.01028	10.57859	10.15368	9.73820	9.33430	8.94368	
23	13.29383	12.93468	12.55133	12.15133	11.74111	11.32609	10.91085	10.49921	10.09430	9.69870	9.31448	8.94327	
22	13.02658	12.69049	12.33124	11.95613	11.57137	11.18221	10.79305	10.40756	10.02876	9.65912	9.30061	8.95478	
21	12.71205	12.40454	12.07514	11.73085	11.37762	11.02047	10.66360	10.31051	9.96407	9.62661	9.29997	8.98562	
20	12.35762	12.08420	11.79040	11.48287	11.16723	10.84824	10.52987	10.21540	9.90753	9.60841	9.31976	9.04289	
19	11.97245	11.73841	11.48570	11.22057	10.94830	10.67334	10.39937	10.12943	9.86598	9.61100	9.36598	9.13209	
18	11.56811	11.37798	11.17113	10.95336	10.72955	10.50377	10.27938	10.05910	9.84513	9.63916	9.44245	9.25592	
17	11.15917	11.01605	10.85849	10.69175	10.52023	10.34756	10.17668	10.00993	9.84913	9.69560	9.55024	9.41361	
16	10.76298	10.66770	10.56081	10.44690	10.32978	10.21250	10.09746	9.98648	9.88085	9.78136	9.68839	9.60190	
15	10.39678	10.34756	10.29048	10.22925	10.16685	10.10559	10.04716	9.99266	9.94269	9.89730	9.85610	9.81821	
14	10.07086	10.06469	10.05544	10.04568	10.03734	10.03180	10.02986	10.03180	10.03734	10.04568	10.05544	10.06469	
13	9.78230	9.81821	9.85610	9.89730	9.94268	9.99266	10.04716	10.10559	10.16685	10.22925	10.29048	10.34756	
12	9.52152	9.60190	9.68838	9.78136	9.88085	9.98648	10.09746	10.21250	10.32978	10.44690	10.56080	10.66770	
11	9.28592	9.41361	9.55024	9.69560	9.84913	10.00993	10.17668	10.34755	10.52023	10.69175	10.85849	11.01605	
10	9.08014	9.25592	9.44245	9.63916	9.84513	10.05910	10.27937	10.50377	10.72955	10.95336	11.17113	11.37798	
9	8.91013	9.13209	9.36598	9.61099	9.86598	10.12943	10.39937	10.67334	10.94830	11.22056	11.48569	11.73840	
8	8.77876	9.04289	9.31976	9.60841	9.90753	10.21540	10.52987	10.84824	11.16723	11.48286	11.79040	12.08420	
7	8.68464	8.98562	9.29997	9.62660	9.96407	10.31051	10.66360	11.02047	11.37762	11.73085	12.07513	12.40453	
6	8.62283	8.95478	9.30061	9.65912	10.02876	10.40756	10.79305	11.18221	11.57137	11.95613	12.33124	12.69049	
5	8.58635	8.94327	9.31448	9.59870	10.09430	10.49921	10.91085	11.32609	11.74110	12.15133	12.55133	12.93468	
4	8.56766	8.94368	9.33430	9.73820	10.15367	10.57859	11.01028	11.44551	11.88035	12.31012	12.72921	13.13105	
3	8.55985	8.94930	9.35358	9.77133	10.20081	10.63982	11.08565	11.53498	11.98381	12.47236	12.85993	13.27482	
2	8.55747	8.95488	9.36727	9.79324	10.23104	10.67842	11.13265	11.59036	12.04751	12.49926	12.93985	13.36250	
1	8.55708	8.95713	9.37220	9.80090	10.24145	10.69161	11.14862	11.60911	12.06902	12.52349	12.96674	13.39196	

J	I =	37	38	39	40	41	42	43	44	45	46	47	48
27	8.55708	8.17311	7.80598	7.45621	7.12414	6.80989	6.51349	6.23480	5.97364	5.72972	5.50271	5.29222	
26	8.55748	8.17609	7.81148	7.46417	7.13448	6.82255	6.52837	6.25183	5.99273	5.75078	5.52564	5.31693	
25	8.55985	8.18626	7.82927	7.48938	7.16690	6.86195	6.57451	6.30446	6.05159	5.81559	5.59612	5.39280	
24	8.56766	8.20723	7.86310	7.53574	7.22543	6.93228	6.65624	6.39717	6.15484	5.92893	5.71908	5.52488	
23	8.58635	8.24464	7.91882	7.60931	7.31636	7.04003	6.78026	6.53687	6.30960	6.09810	5.90199	5.72084	
22	8.62283	8.30562	8.00377	7.71764	7.44743	7.19316	6.95471	6.73188	6.52436	6.33177	6.15369	5.98966	
21	8.68464	8.39783	8.12570	7.86856	7.62655	7.39961	7.18758	6.99020	6.80711	6.63789	6.48207	6.33917	
20	8.77877	8.52807	8.29123	8.06847	7.85980	7.66513	7.48420	7.31670	7.16220	7.02025	6.89032	6.77189	
19	8.91013	8.70063	8.50387	8.31995	8.14878	7.99014	7.84369	7.70903	7.58568	7.47313	7.37083	7.27824	
18	9.08014	8.91542	8.76184	8.61928	8.48747	8.36601	8.25444	8.15222	8.05880	7.97364	7.89623	7.82607	
17	9.28592	9.16710	9.05687	8.95474	8.86007	8.77214	8.69017	8.61338	8.54106	8.47260	8.40759	8.34580	
16	9.52152	9.44654	9.37598	9.30861	9.24303	9.17771	9.11106	9.04158	8.96789	8.88897	8.80428	8.71398	
15	9.78230	9.74661	9.70893	9.66663	9.61672	9.55583	9.48032	9.38634	9.27006	9.12792	8.95704	8.75593	
14	10.07085	10.07073	10.06038	10.03510	9.98931	9.91646	9.80887	9.65758	9.45212	9.18023	8.82743	8.37651	
13	10.39677	10.43355	10.45238	10.44670	10.40868	10.32909	10.19703	9.99959	9.72155	9.34480	8.84778	8.20461	
12	10.76298	10.84109	10.89546	10.91829	10.90047	10.83131	10.69840	10.48733	10.18141	9.76141	9.20523	8.48757	
11	11.15917	11.28159	11.37596	11.43367	11.44470	11.39744	11.27849	11.07243	10.76159	10.32581	9.74211	8.98445	
10	11.56811	11.73470	11.86976	11.96398	12.00656	11.98505	11.88508	11.69020	11.38158	10.93773	10.33418	9.54316	
9	11.97245	12.18050	12.35398	12.48295	12.55588	12.55946	12.47840	12.29517	11.98968	11.53904	10.91713	10.09430	
8	12.35762	12.60286	12.81085	12.97103	13.07119	13.09725	13.03303	12.85995	12.55677	12.09922	11.45967	10.60665	
7	12.71205	12.98949	13.22730	13.41441	13.53798	13.58325	13.53321	13.36837	13.06644	12.60193	11.94583	11.06511	
6	13.02658	13.33097	13.59371	13.80327	13.94628	14.00737	13.96884	13.81041	13.50887	13.03771	12.36672	11.46154	
5	13.29383	13.61996	13.90277	14.13036	14.28892	14.36258	14.33307	14.17944	13.87774	13.40060	12.71683	11.79096	
4	13.50789	13.85067	14.14884	14.39020	14.56060	14.64378	14.62100	14.47081	14.16866	13.68653	12.99247	12.05012	
3	13.66413	14.01865	14.32765	14.57869	14.75740	14.84721	14.82908	14.68119	14.37856	13.89268	13.19107	12.23675	
2	13.75922	14.12072	14.43615	14.69294	14.87657	14.97030	14.95489	14.80831	14.50532	14.01712	13.31090	12.34931	
1	13.79115	14.15496	14.47252	14.73121	14.91647	15.01150	14.99699	14.85083	14.54771	14.05873	13.35096	12.38693	

J	I =	49	50	51	52	53	54	55	56	57	58	59	60
27	5.09787	4.91923	4.75588	4.57948	4.42379	4.28813	4.17189	4.07452	3.99554	3.93453	3.89119	3.86528	
26	5.12426	4.94719	4.78531	4.61054	4.45631	4.32195	4.20685	4.11044	4.03225	3.97187	3.92897	3.90332	
25	5.20521	5.03292	4.87549	4.70563	4.55584	4.42543	4.31377	4.22030	4.14451	4.08601	4.04446	4.01962	
24	5.34590	5.18170	5.03184	4.87033	4.72807	4.60436	4.49855	4.41005	4.33836	4.28306	4.24380	4.22034	
23	5.55519	5.40158	5.26254	5.11299	4.98153	4.86742	4.77000	4.68865	4.62283	4.57212	4.53614	4.51466	
22	5.83919	5.70179	5.57696	5.44310	5.32582	5.22433	5.13792	5.06594	5.00785	4.96316	4.93151	4.91263	
21	6.20864	6.08998	5.98264	5.86809	5.76823	5.68223	5.60935	5.54890	5.50028	5.46300	5.43666	5.42097	
20	6.66440	6.56729	6.48002	6.38757	6.30760	6.23927	6.18178	6.13443	6.09659	6.06773	6.04743	6.03537	
19	7.19483	7.12004	7.05336	6.98337	6.92350	6.87292	6.83087	6.79666	6.76964	6.74924	6.73501	6.72661	
18	7.76277	7.70595	7.65529	7.60224	7.55716	7.51953	7.48875	7.46420	7.44525	7.43125	7.42169	7.41613	
17	8.28726	8.23222	8.18114	8.12579	8.07780	8.03761	8.00516	7.97995	7.96115	7.94783	7.93907	7.93414	
16	8.61913	8.52186	8.42533	8.31570	8.21919	8.13919	8.07630	8.02919	7.99553	7.97271	7.95832	7.95048	
15	8.52554	8.27093	8.00388	7.69694	7.44275	7.24839	7.10775	7.01044	6.94586	6.90491	6.88053	6.86782	
14	7.80673	7.09268	6.20258	5.17238	4.54691	4.17153	3.94929	3.82002	3.74660	3.70625	3.68518	3.67532	
13	7.38402	6.34787	5.04923	3.48522	2.66384	2.23362	2.01039	1.89678	1.84089	1.81485	1.80370	1.79952	
12	7.57967	6.44910	5.05978	3.39287	2.51138	2.05091	1.81508	1.69811	1.64304	1.61923	1.61028	1.60762	
11	8.02335	6.82566	5.35415	3.58903	2.65621	2.16997	1.92216	1.80047	1.74426	1.72087	1.71278	1.71081	
10	8.53318	7.26861	5.70919	3.83669	2.84863	2.33480	2.07397	1.94677	1.88879	1.86530	1.85769	1.85619	
9	9.03683	7.70654	6.06018	4.08114	3.03798	2.49643	2.22231	2.08931	2.02926	2.00541	1.99807	1.99692	
8	9.50443	8.11249	6.38491	4.30636	3.21155	2.64385	2.35707	2.21843	2.15626	2.13193	2.12475	2.12387	
7	9.92225	8.47467	6.67417	4.50638	3.36511	2.77382	2.47555	2.33173	2.26756	2.24273	2.23563	2.23495	
6	10.28310	8.78712	6.92340	4.67835	3.49682	2.88502	2.57673	2.42835	2.36240	2.33709	2.33004	2.32951	
5	10.58269	9.04630	7.12998	4.82068	3.60563	2.97674	2.66008	2.50788	2.44040	2.41467	2.40764	2.40723	
4	10.81822	9.24992	7.29217	4.93230	3.69086	3.04850	2.72523	2.57000	2.50131	2.47524	2.46821	2.46788	
3	10.98774	9.39641	7.40880	5.01252	3.75205	3.09999	2.77194	2.61452	2.54495	2.51861	2.51158	2.51132	
2	11.08995	9.48471	7.47909	5.06083	3.78889	3.13096	2.80003	2.64129	2.57118	2.54469	2.53766	2.53743	
1	11.12411	9.51421	7.50257	5.07697	3.80019	3.14130	2.80941	2.65022	2.57994	2.55339	2.54635	2.54614	

J I = 61
27 3.85665
26 3.89479
25 4.01136
24 4.21254
23 4.50752
22 4.90636
21 5.41576
20 6.03137
19 6.72384
18 7.41431
17 7.93255
16 7.94800
15 6.86390
14 3.67247
13 1.79850
12 1.60713
11 1.71059
10 1.85615
9 1.99703
8 2.12408
7 2.23525
6 2.32989
5 2.40766
4 2.46836
3 2.51182
2 2.53795
1 2.54666

BENCHMARK PROBLEM SOLUTION

Identification: 4-A1-2

Benchmark Problem ID.4-A1

Date Submitted: December 15, 1967 By: Eugene Wachspress (KAPL)
(Name and Organization)Date Adopted: December 1967 By: H. Greenspan (ANL)
(Name and Organization)Descriptive Title: Flux Synthesis Computation of a Nonseparable Reactor
Theory

Nodal equations were obtained by the variational prescription in Refs. 1-3. These equations were solved for the fundamental flux mode by Wielandt's method as described in Ref. 4. The trial-functions were obtained by solving two 1-D problems:

Trial Function 1 is the fundamental flux mode for the lower half of the configuration in 4-A1.

Trial Function 2 is the fundamental flux mode for the upper half of the configuration in 4-A1.

These fluxes were used as both the direct- and adjoint-trial-functions (Galerkin's method).

Computations were performed with one and with two trial-functions per axial zone.

The number of radial nodes (or channels) was varied in this study.

Computer: CDC-6600

Date Solved: October 1966

at: KAPL
(Installation)

Program: 2D- $\nu\nabla$. This was an experimental program and is no longer available.

Variational Mockup1. Trial-functions

When only one trial-function was used in each axial zone (that is, $M = 1$), trial-function No. 1 was taken for the lower zone and No. 2 was used in the upper zone.

When two trial-functions were used in each axial zone (that is, $M = 2$), both trial-functions were used throughout.

2. Radial Nodes

Coefficients multiplying the trial-functions are constant over sets of meshpoints defined as follows:

	No. of Channels, N				
	1	3	5	9	12
Intervals of channels; meshpoint i_1 to i_2 is denoted by (i_1, i_2) .	(1,11) (1,61)	(1,6) (11,51)	(1,6)(6,11)(11,16) (6,11) (11,51)	(1,6)(6,11)(11,16) (16,21)(21,41)(41,46)	(1,6)(6,11) (11,16)(16,21)(21,26) (26,31)(31,36)(36,41) (41,46)(46,51)(51,56) (56,61)

3. Synthesized flux solution

Index g = energy group index ($g = 1, 2, 3, 4$)

n = channel index ($n = 1, 2, \dots, N$)
where N = total number of channels

m = trial function index

$m = 1$ is trial-function No. 1

$m = 2$ is trial-function No. 2

$\Phi_m(x) =$ flux shape for trial-function m

$a_{gmn}(y) =$ coefficient of trial-function m at elevation y in channel n and energy group g

$\Phi_g(x,y) =$ flux at point (x,y) in energy group g

$M = 1$ means only one trial-function was used per zone

$M = 2$ means both trial-functions were in each zone

The synthesized flux was for (x,y) at meshpoint (i,j) :

$$M = 1: \left\{ \begin{array}{l} \Phi_g(x,y) = a_{g1n}(j)\Phi_{g1}(i) \\ \text{lower half} \\ (y < 22.86 \text{ cm}) \\ \Phi_g(x,y) = a_{g2n}(j)\Phi_{g2}(i) \\ \text{upper half} \\ (y > 22.86 \text{ cm}) \\ \Phi_g(x,y) \text{ is discontinuous on line } y = 22.86 \text{ cm } (j = 14) \end{array} \right.$$

$$M = 2: \Phi_g(x,y) = a_{g1n}(j)\Phi_{g1}(i) + a_{g2n}(j)\Phi_{g2}(i)$$

For example, coefficient $a_{214}(6)$ is the coefficient in channel $n = 4$ on line $j = 6$ for the group $g = 2$ flux of trial-function No. 1. If $N = 9$ (a nine-channel problem), then $n = 4$ is the interval $i = 16$ to $i = 21$. The coefficients $a_{gmn}(z)$ are not displayed. The synthesized flux, $\Phi_g(x,y)$, is edited output of 2D- $\nu\bar{V}$.

Primary Results

N	M	Eigenvalue k (Correct to four significant digits)	Percent Error From 2-D NOVA Result of k = 0.8791
3	1	0.8891	1.1
9	1	0.8833	0.48
12	1	0.8851	0.68
1	2	0.9482	7.29
5	2	0.8787	0.04
12	2	0.8791	-

Secondary Results

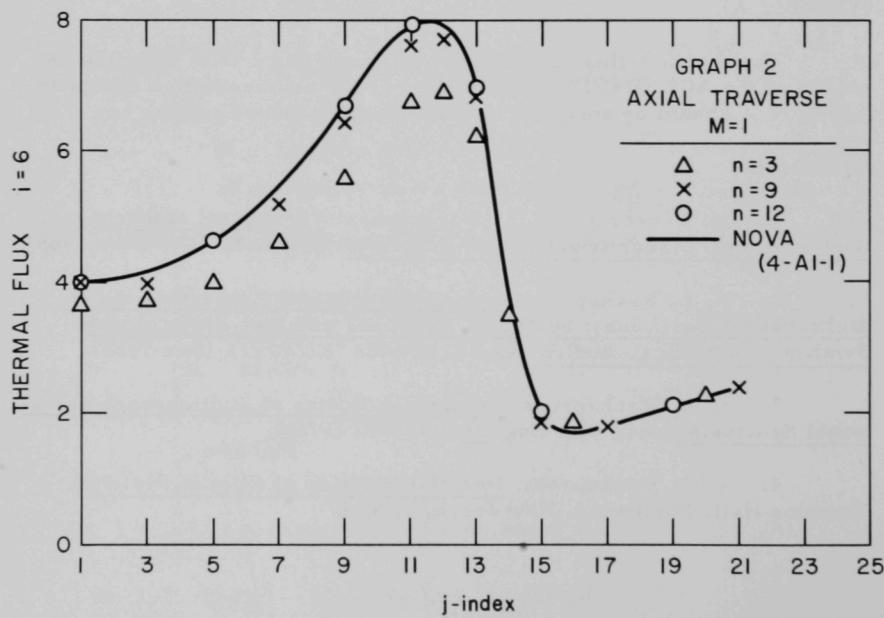
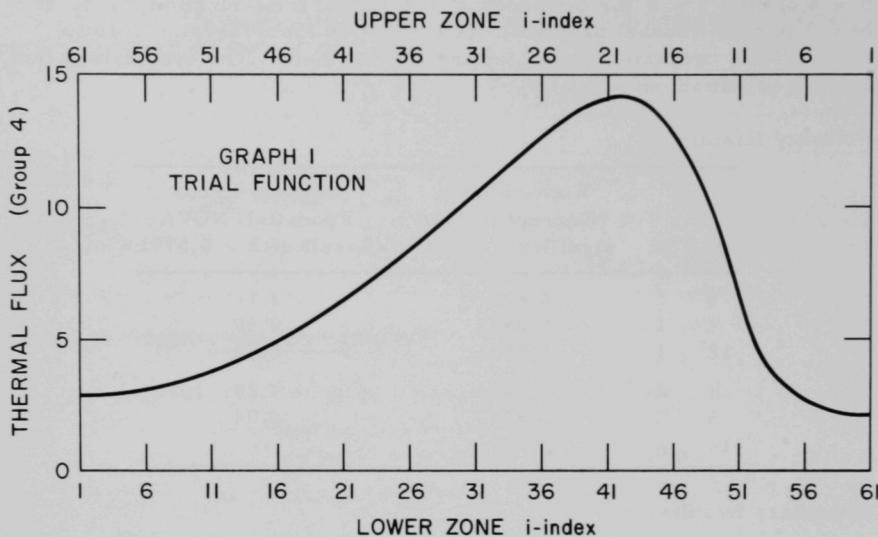
Thermal-flux traverses are shown in the attached graphs (EXHIBIT A).

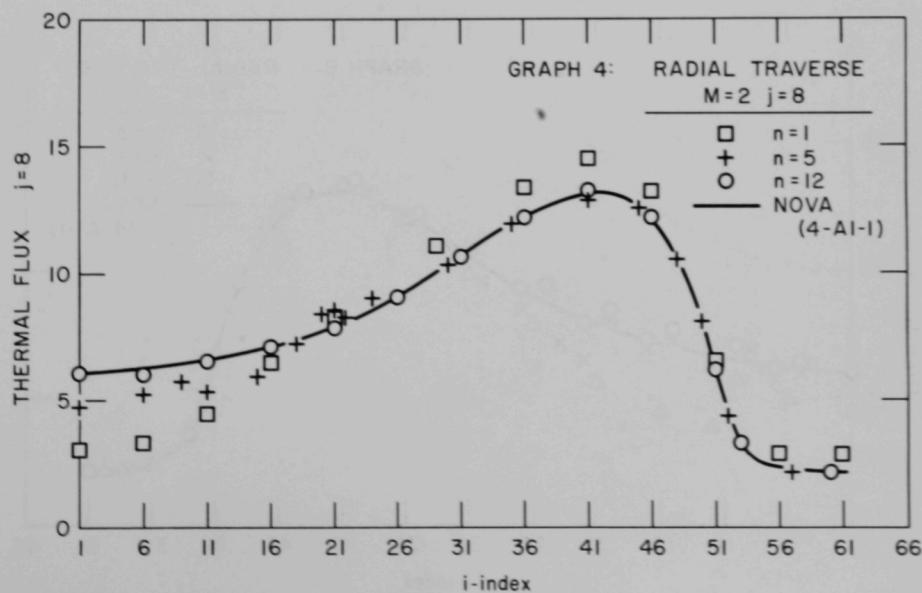
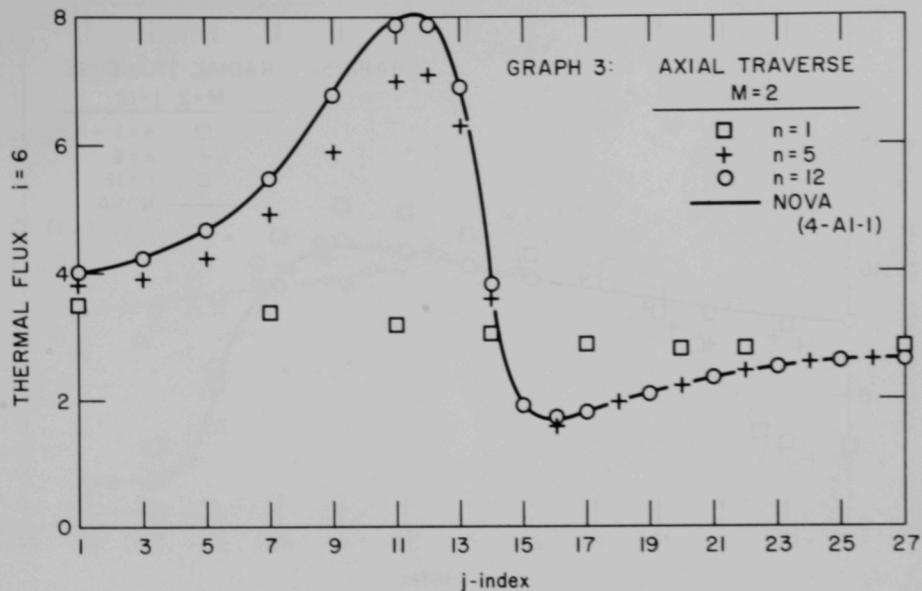
The thermal-flux output for 12 channels and 2 trial functions is on file at the ACC (EXHIBIT B). However, the representative values in EXHIBIT A should be adequate for evaluation purposes.

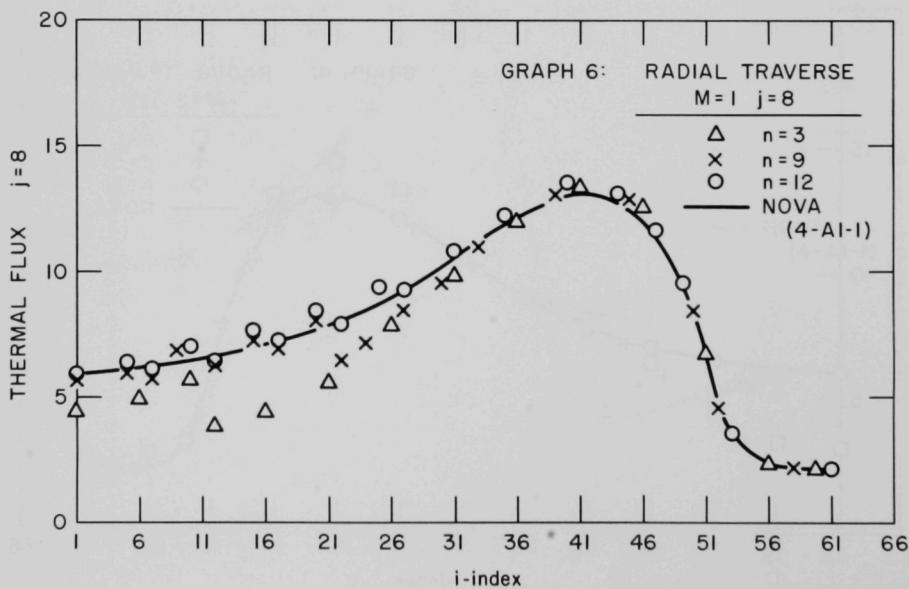
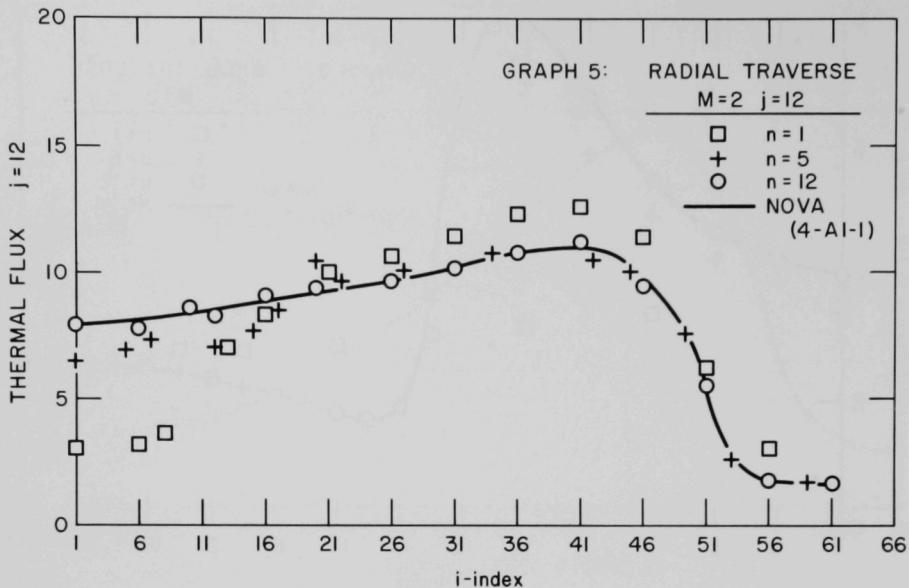
References

1. M. Becker and E. L. Wachspress, Variational Multichannel Synthesis with Discontinuous Trial Functions, KAPL-M-3095 (Nov 1965).
2. E. L. Wachspress, Some Mathematical Properties of the Multichannel Variational Synthesis Equations and Two-dimensional Synthesis Numerical Studies, KAPL-M-6588 (ELW-13), (Dec 1966).
3. E. L. Wachspress, Numerical Studies of Multichannel Variational Synthesis, Nucl. Sci. Eng. 26, 373-377 (1966).
4. E. L. Wachspress, Iterative Solution of Elliptic Systems, Prentice Hall, Englewood, New Jersey (1966).

EXHIBIT A







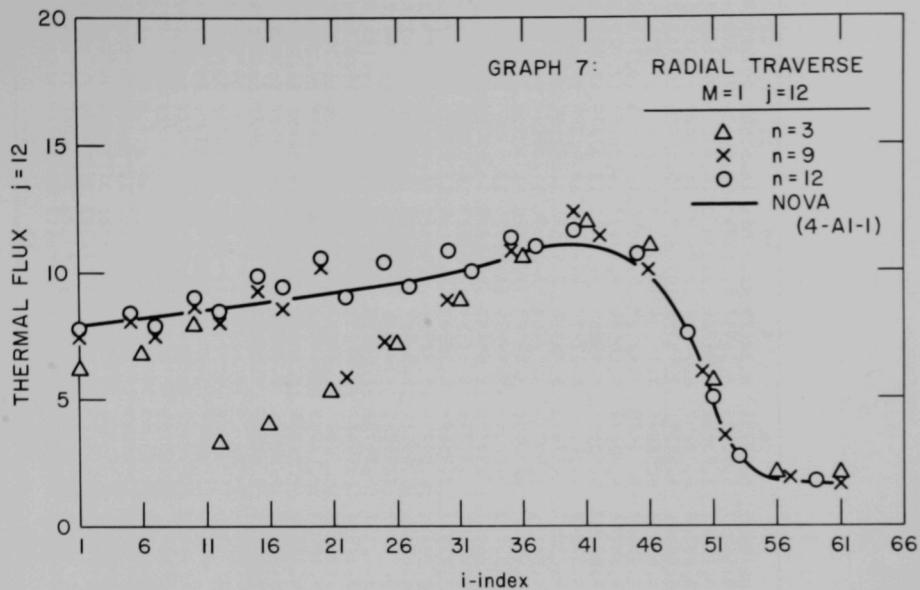


EXHIBIT B

Group 4: Thermal Group--Topological Flux Edit

J	I =	1	2	3	4	5	6	7	8	9	10	11	12
27		5.61586	5.61369	5.61155	5.62411	5.68208	5.83710	6.18899	6.92791	8.38679	11.19545	16.53580	20.96077
26		5.59657	5.59444	5.59236	5.60497	5.66282	5.81746	6.16837	6.90505	8.35957	11.15996	16.48430	20.89654
25		5.53877	5.53673	5.53486	5.54761	5.60512	5.75860	6.10659	6.83654	8.27805	11.05370	16.33011	20.70432
24		5.44258	5.44069	5.43919	5.45218	5.50912	5.66071	6.00388	6.72267	8.14262	10.87729	16.07429	20.38555
23		5.30824	5.30657	5.30560	5.31896	5.37513	5.52411	5.86061	6.56391	7.95395	10.63181	15.71860	19.94269
22		5.13805	5.13468	5.13442	5.14830	5.20355	5.34926	5.67735	6.36095	7.71304	10.31883	15.26570	19.37939
21		4.92647	4.92548	4.92615	4.94076	4.99494	5.13683	5.45489	6.11479	7.42129	9.94066	14.71941	18.70092
20		4.68014	4.67964	4.68151	4.69712	4.75020	4.88782	5.19446	5.82694	7.0886	9.50077	14.08557	17.91523
19		4.39844	4.39857	4.40200	4.41901	4.47108	4.60418	4.89832	5.50017	6.69560	9.00512	13.37399	17.03560
18		4.04589	4.08682	4.09229	4.11129	4.16264	4.29130	4.57246	5.14135	6.27420	8.46607	12.60398	16.08592
17		3.76265	3.76460	3.77269	3.79448	3.84574	3.97045	4.23897	4.77442	5.84396	7.91695	11.82145	15.12033
16		3.53245	3.53547	3.54637	3.57146	3.62390	3.74505	4.00181	4.50750	5.51768	7.47571	11.16151	14.27730
15		3.95281	3.95594	3.96747	3.99461	4.05241	4.17113	4.41729	4.90638	5.85585	7.66568	11.10332	13.97219
14		8.07387	8.07056	8.06692	8.08420	8.16682	8.35583	8.65973	9.18068	9.99275	11.31356	13.60158	15.40973
13		15.11430	15.07841	14.98837	14.90396	14.95017	15.14941	15.44087	15.99792	16.55681	17.01856	17.47902	17.85452
12		17.50078	17.43916	17.27870	17.10210	17.08235	17.21735	17.43321	17.97983	18.41134	18.51627	18.37224	18.34353
11		17.44716	17.37824	17.19731	16.99182	16.94480	17.05008	17.23032	17.75061	18.12924	18.13136	17.83067	17.70920
10		16.28526	16.22204	16.05626	15.86880	15.82896	15.93180	16.10494	16.59256	16.94961	16.95827	16.68403	16.58912
9		14.75169	14.70013	14.56593	14.41855	14.40328	14.51687	14.69654	15.15116	15.50009	15.55704	15.37728	15.35166
8		13.22202	13.18354	13.08494	12.98327	12.99785	13.12663	13.31782	13.74250	14.08916	14.20541	14.13641	14.18939
7		11.86755	11.84128	11.77596	11.71721	11.76049	11.90457	12.10797	12.50732	12.85425	13.02735	13.06275	13.18919
6		10.75028	10.73438	10.69734	10.67517	10.74321	10.90080	11.11524	11.49427	11.84251	12.06461	12.18915	12.37824
5		9.87769	9.87005	9.85554	9.86256	9.95053	10.11915	10.34275	10.70623	11.05613	11.31768	11.51355	11.75288
4		9.23357	9.23212	9.23449	9.26341	9.36642	9.54347	9.77413	10.12633	10.47783	10.76922	11.01878	11.29602
3		8.79492	8.79773	8.81173	8.85576	8.96918	9.15212	9.38778	9.73239	10.08519	10.39731	10.68400	10.98753
2		8.54101	8.54630	8.56709	8.61994	8.73945	8.92587	9.16450	9.50476	9.85840	10.18269	10.49112	10.81068
1		8.45797	8.46409	8.48710	8.54285	8.66437	8.85194	9.09155	9.43040	9.78433	10.11262	10.42821	10.75224

J	I =	13	14	15	16	17	18	19	20	21	22	23	24
27	24.50316	27.29343	29.43436	31.00182	32.08061	32.75829	33.08291	33.10975	32.88985	32.49273	31.92651	31.22559	
26	24.42897	27.21129	29.34600	30.90962	31.98675	32.66372	32.98922	33.01829	32.80159	32.40769	31.84562	31.14945	
25	24.20700	26.96555	29.08166	30.63381	31.70605	32.38096	32.70916	32.74502	32.53796	32.15382	31.60429	30.92244	
24	23.83902	26.55825	28.64355	30.17683	31.24121	31.91290	32.24585	32.29329	32.10259	31.73489	31.20649	30.54877	
23	23.32408	25.99288	28.03549	29.54286	30.59682	31.26442	31.60452	31.66874	31.50149	31.15721	30.65885	30.03541	
22	22.67872	25.27464	27.26315	28.73807	29.77961	30.44271	30.79283	30.87953	30.74333	30.42977	29.97075	29.39210	
21	21.89746	24.44109	26.33466	27.77130	28.79920	29.45786	29.82141	29.93687	29.83989	29.56462	29.15458	28.63159	
20	20.99406	23.41307	25.26216	26.65552	27.66953	28.32442	28.70535	28.85635	28.80715	28.57788	28.22658	27.77025	
19	19.94438	22.249886	24.00517	25.41180	26.41234	27.06461	27.46716	27.66061	27.66768	27.49175	27.20859	26.82939	
18	18.89718	21.10046	22.77834	24.07594	25.06419	25.71509	26.14288	26.38444	26.45461	26.33821	26.13105	25.83777	
17	17.79129	19.88130	21.46916	22.71602	23.69074	24.33998	25.79311	25.08323	25.21726	25.16422	25.03733	24.83465	
16	16.80160	18.77660	20.27678	21.46490	22.41090	23.05188	23.51897	23.84209	24.02420	24.03562	23.98632	23.87123	
15	16.26292	18.07575	19.47634	20.56407	21.41512	22.02442	22.46617	22.76913	22.94893	23.02880	23.04446	23.02844	
14	16.93231	18.26122	19.42851	20.28589	20.86708	21.39948	21.74199	21.92582	22.01926	22.18813	22.24998	22.26091	
13	18.22564	18.43501	19.66380	20.24295	20.49462	20.93823	21.16092	21.19746	21.18215	21.44848	21.55174	21.60977	
12	18.27837	18.0628	19.28992	19.73434	19.83982	20.24152	20.41732	20.40230	20.35719	20.69968	20.86169	20.98577	
11	17.53780	17.78190	18.40111	18.80422	18.89435	19.29662	19.49230	19.51380	19.51815	19.92662	20.16675	20.37845	
10	16.44778	16.68928	17.27655	17.68195	17.81976	18.24500	18.49776	18.60316	18.70136	19.17264	19.50155	19.81169	
9	15.29046	15.56042	16.13005	16.55518	16.76380	17.22344	17.54440	17.75009	17.95728	18.48832	18.90704	19.31697	
8	14.21949	14.52690	15.08599	15.53596	15.81838	16.31290	16.70297	17.00752	17.32120	17.90718	18.40981	18.91280	
7	13.30473	13.64895	14.20155	14.67596	15.02578	15.55270	16.00521	16.39835	16.80717	17.44168	18.01835	18.60337	
6	12.56666	12.94314	13.49183	13.98786	14.39494	14.94994	15.45546	15.92328	16.41222	17.08795	17.72705	18.38110	
5	11.99967	12.40247	12.94896	13.46292	13.91603	14.49409	15.04235	15.57005	16.12317	16.83248	17.52191	18.23156	
4	11.58683	12.00977	12.55518	13.08307	13.57110	14.16701	14.74783	15.32091	15.92267	16.65789	17.38577	18.13782	
3	11.30885	11.74592	12.29088	12.82867	13.34106	13.94963	14.55326	15.15798	15.79365	16.54722	17.30209	18.08390	
2	11.14928	11.59470	12.13954	12.68324	13.20997	13.82610	14.44321	15.06657	15.72221	16.48672	17.25758	18.05700	
1	11.09733	11.54551	12.09033	12.63599	13.16746	13.78610	14.40767	15.03719	15.69942	16.46757	17.24372	18.04897	

J	I =	25	26	27	28	29	30	31	32	33	34	35	36
27	30.41757	29.52634	28.54680	27.62334	26.62093	25.60231	24.57752	23.57753	22.58916	21.61766	20.666780	19.74520	
26	30.34667	29.46110	28.53723	27.56966	26.57328	25.56080	24.54210	23.54832	22.56618	21.60082	20.65695	19.74020	
25	30.13547	29.26695	28.30019	27.41034	26.43217	25.43819	24.43785	23.46277	22.49946	21.55260	20.62678	19.72772	
24	29.78839	28.94856	28.07057	27.15053	26.20298	25.24012	24.27065	23.32706	22.39545	21.47978	20.58436	19.71507	
23	29.31276	28.51361	27.67642	26.79862	25.89446	24.97572	24.04998	23.15108	22.26449	21.39309	20.54083	19.71382	
22	28.71873	27.97269	27.18870	26.36597	25.51837	24.65713	23.78838	22.94782	22.12001	21.30641	20.51051	19.73872	
21	28.01940	27.33924	26.62112	25.86658	25.08900	24.29894	23.50072	22.73242	21.97744	21.23545	20.50937	19.80660	
20	27.23121	26.52978	25.99017	25.31687	24.62271	23.91746	23.20329	22.52101	21.85272	21.19600	20.55304	19.93110	
19	26.37493	25.56446	25.31533	24.73565	24.13760	23.53007	22.91284	22.32942	21.76069	21.20187	20.65435	20.12585	
18	25.47740	25.06804	24.61973	24.14434	23.65336	23.15469	22.64577	22.17204	21.71348	21.26301	20.82108	20.39584	
17	24.57345	24.27058	23.93053	23.56714	23.19114	22.80937	22.41754	22.06128	21.71995	21.38478	21.05523	20.73946	
16	23.70585	23.50567	23.27725	23.02961	22.77243	22.51147	22.24219	22.00790	21.78674	21.56976	21.35539	21.15086	
15	22.91798	22.40296	22.68523	22.55356	22.41542	22.27548	22.13147	22.02168	21.912079	21.82208	21.72297	21.62755	
14	22.23343	22.17669	22.16604	22.14976	22.13005	22.11053	22.09342	22.11045	22.12988	22.14951	22.16571	22.17629	
13	21.63447	21.62771	21.72306	21.82208	21.92071	22.02151	22.13121	22.27515	22.41500	22.55306	22.68465	22.80230	
12	21.08255	21.15078	21.35523	21.56951	21.78642	22.00748	22.24170	22.51089	22.77177	23.02886	23.27642	23.50476	
11	20.56967	20.73915	21.05484	21.38430	21.71939	22.06063	22.41680	22.80855	23.19023	23.56615	23.92946	24.26943	
10	20.11057	20.39531	20.82047	21.26231	21.71269	22.17117	22.64481	23.15364	23.65222	24.14312	24.61843	25.06666	
9	19.72399	20.12512	20.65353	21.20097	21.75970	22.32834	22.91168	23.52881	24.13625	24.73421	25.31381	25.86285	
8	19.42058	19.93019	20.55205	21.19492	21.85156	22.51975	23.20194	23.91602	24.62117	25.31525	25.98845	26.62798	
7	19.19986	19.80494	20.50822	21.23421	21.97611	22.73100	23.49921	24.29733	25.08729	25.86478	26.61923	27.33726	
6	19.05220	19.73752	20.50923	21.30503	22.11855	22.94626	23.78672	24.65537	25.51652	26.36403	27.18665	27.97056	
5	18.96269	19.71252	20.53944	21.39160	22.26292	23.14941	24.04820	24.97384	25.89249	26.79655	27.67426	28.51135	
4	18.91471	19.71369	20.58288	21.47821	22.39379	23.32529	24.26878	25.23815	26.20091	27.14836	28.06831	28.94620	
3	18.89247	19.72624	20.62525	21.55097	22.49773	23.46094	24.43592	25.43616	26.43003	27.40810	28.35785	29.26452	
2	18.88495	19.73872	20.65537	21.59915	22.56442	23.54645	24.54013	25.55873	26.57111	27.56738	28.53486	29.45863	
1	18.88320	19.74371	20.66622	21.61598	22.58738	23.57565	24.57554	25.60022	26.61874	27.62105	28.59441	29.52386	

J	I =	37	38	39	40	41	42	43	44	45	46	47	48
27	18.88460	18.05028	17.24495	16.46871	15.70049	15.03818	14.40860	13.78697	13.16827	12.63674	12.09103	11.54617	
26	18.88634	18.05830	17.25880	16.48786	15.72327	15.06756	14.44412	13.82695	13.21076	12.68397	12.14023	11.59535	
25	18.89422	18.08516	17.30327	16.54832	15.79468	15.15894	14.55415	13.95045	13.34182	12.82938	12.29154	11.74654	
24	18.91600	18.13903	17.38689	16.65894	15.92365	15.32181	14.74866	14.16777	13.57181	13.08372	12.55579	12.01034	
23	18.96390	18.23268	17.52295	16.83345	16.12406	15.57087	15.04310	14.49478	13.91667	13.6350	12.94949	12.40297	
22	19.05331	18.38212	17.72798	17.08881	16.41301	15.92400	15.45610	14.95052	14.39548	13.98834	13.49226	12.94354	
21	19.20083	18.60425	18.01916	17.44241	16.80784	16.39894	16.00573	15.55315	15.02619	14.67631	14.20186	13.64924	
20	19.42140	18.91354	18.41046	17.90776	17.32172	17.00796	16.70333	16.31321	15.81864	15.53616	15.08615	14.52704	
19	19.72463	19.31753	18.90752	18.48872	17.95762	17.75036	17.54459	17.2357	16.76389	16.55521	16.13004	15.56039	
18	20.11101	19.81205	19.50183	19.17284	18.70151	18.60322	18.49775	18.24593	17.81965	17.68179	17.27635	16.68906	
17	20.56490	20.37819	20.16681	19.92662	19.51809	19.51365	19.49208	19.29634	18.89404	18.80385	18.40071	17.78149	
16	21.04254	20.98568	20.80153	20.69944	20.35690	20.40193	20.41688	20.24102	19.83929	19.73376	19.28932	18.60568	
15	21.63422	21.60945	21.55134	21.44801	21.18163	21.19687	21.16027	20.93753	20.49390	20.24218	19.66303	18.83426	
14	22.23334	22.26034	22.24934	22.18743	22.01851	21.92501	21.74112	21.39858	20.86618	20.28497	19.42761	18.26036	
13	22.91124	23.00203	23.04359	23.02786	22.94795	22.76810	22.46509	22.02332	21.41402	20.56298	19.47529	18.07477	
12	23.70447	23.87017	23.98520	24.03444	24.02298	23.84082	23.51766	23.05056	22.40960	21.46362	20.27555	18.77546	
11	24.57222	24.83335	25.03596	25.16279	25.21580	25.08172	24.79158	24.33844	23.68921	22.71453	21.46773	19.87998	
10	25.47593	25.83623	26.12945	26.33655	26.45290	26.38270	26.14111	25.71332	25.06244	24.07424	22.77671	21.09895	
9	26.37325	26.52763	27.20676	27.48987	27.65575	27.65865	27.46518	27.06263	26.41039	25.40990	24.06336	22.29718	
8	27.22933	27.76829	28.22455	28.57579	28.80502	28.85419	28.70317	28.32224	27.66739	26.65354	25.26017	23.41122	
7	28.01734	28.52945	29.15236	29.56235	29.83757	29.93452	29.81905	29.45550	28.79688	27.76905	26.33252	24.40900	
6	28.71651	29.38980	29.96838	30.42734	30.74086	30.87702	30.79030	30.44019	29.77715	28.73567	27.26087	25.27252	
5	29.31041	30.03294	30.65635	31.15465	31.49888	31.56610	31.60187	31.26178	30.59422	29.54034	28.03310	25.99066	
4	29.78594	30.54624	31.20388	31.73222	32.09988	32.29055	32.24309	31.91015	31.23852	30.17422	28.64106	26.55594	
3	30.13295	30.41984	31.60161	32.15107	32.53518	32.74220	32.70633	32.37815	31.70329	30.63113	29.07911	26.96318	
2	30.34411	31.14680	31.84290	32.40490	32.79875	33.01542	32.98634	32.66086	31.98394	30.90689	29.34341	27.20889	
1	30.41499	31.22292	31.92377	32.48993	32.88701	33.10687	33.08002	32.75542	32.07779	30.99909	29.43176	27.29101	

J	I	=	49	50	51	52	53	54	55	56	57	58	59	60
27	11.09795	10.75281	10.42873	10.11310	9.78476	9.43081	9.09194	8.85231	8.66472	8.54318	8.48742	8.46440		
26	11.14988	10.81064	10.49164	10.18316	9.85883	9.50516	9.16488	8.92623	8.73980	8.62026	8.56740	8.54660		
25	11.30942	10.98805	10.68449	10.39774	10.08558	9.73276	9.38813	9.15245	8.96949	8.85605	8.81201	8.79800		
24	11.58735	11.29649	11.01922	10.76960	10.47717	10.12664	9.77443	9.54374	9.36668	9.26366	9.23471	9.23234		
23	12.00013	11.75328	11.51391	11.31798	11.05639	10.70647	10.34297	10.11936	9.95072	9.86273	9.85569	9.87020		
22	12.56702	12.37855	12.18941	12.06481	11.84268	11.49441	11.11537	10.90091	10.74330	10.67524	10.69739	10.73442		
21	13.30497	13.18937	13.06289	13.02743	12.85528	12.50734	12.10797	11.90456	11.76045	11.71716	11.77589	11.84120		
20	14.21959	14.18943	14.13640	14.20533	14.08903	13.74236	13.31768	13.12647	12.99767	12.98307	13.08472	13.18330		
19	15.29039	15.35154	15.37710	15.55678	15.49979	15.15085	14.69623	14.51653	14.40294	14.41818	14.56554	14.69971		
18	16.44753	16.58880	16.68365	16.95782	16.94911	16.59205	16.10444	15.93129	15.82843	15.86825	16.05568	16.22144		
17	17.53736	17.70870	17.83011	18.13072	18.12856	17.74992	17.22964	17.04939	16.94409	16.99109	17.19655	17.37746		
16	18.27776	18.34286	18.37151	18.51548	18.41052	17.97900	17.43240	17.21654	17.08153	17.10126	17.27783	17.43827		
15	18.22489	17.85374	17.47822	17.01773	16.55398	15.99710	15.44006	15.14861	14.94937	14.90315	14.98754	15.07755		
14	16.93150	15.40896	13.60088	11.31296	9.99219	9.18016	8.65923	8.35534	8.16633	8.08371	8.06642	8.07006		
13	16.26203	13.97140	11.10268	7.66522	5.85549	4.90607	4.41700	4.17085	4.05214	3.99434	3.96719	3.95566		
12	16.80057	14.27641	11.16080	7.47522	5.51730	4.50718	4.00152	3.74478	3.62363	3.57120	3.54610	3.53520		
11	17.79010	15.11930	11.82063	7.91638	5.84353	4.77406	4.23864	3.97015	3.84545	3.79418	3.77239	3.76430		
10	18.89583	16.08475	12.60305	8.46543	6.27372	5.14094	4.57210	4.29096	4.16230	4.11096	4.09196	4.08649		
9	19.94287	17.03410	13.37296	9.00441	6.69506	5.49972	4.89792	4.60380	4.47070	4.41864	4.40163	4.39820		
8	20.99240	17.91381	14.08443	9.50000	7.08028	5.82645	5.19402	4.88741	4.74980	4.69672	4.68111	4.67924		
7	21.89567	18.69938	14.71819	9.93983	7.42066	6.11427	5.45443	5.13639	4.99451	4.94033	4.92572	4.92505		
6	22.67681	19.37775	15.26440	10.31795	7.71237	6.36040	5.67686	5.34879	5.20309	5.14785	5.13397	5.13423		
5	23.32608	19.94098	15.71724	10.63088	7.95326	6.56333	5.86009	5.52362	5.37465	5.31848	5.30512	5.30609		
4	23.83694	20.38378	16.07288	10.87633	8.14190	6.72206	6.00334	5.66020	5.50863	5.45169	5.43870	5.44020		
3	24.20487	20.70250	16.32867	11.05271	8.27731	6.83592	6.10604	5.75808	5.60461	5.54710	5.53436	5.53622		
2	24.42681	20.89469	16.48283	11.15896	8.35882	6.90442	6.16781	5.81693	5.66231	5.60446	5.59185	5.59393		
1	24.50099	20.95891	16.53433	11.19445	8.38603	6.92728	6.18843	5.83657	5.68156	5.62359	5.61103	5.61378		

J	I =
27	61 8.45828
26	8.54130
25	8.79519
24	9.23379
23	9.87783
22	10.75032
21	11.86746
20	13.22177
19	14.75127
18	16.28465
17	17.44637
16	17.49988
15	15.11344
14	8.07337
13	3.95253
12	3.53218
11	3.76235
10	4.08555
9	4.39807
8	4.67973
7	4.92603
6	5.13560
5	5.30776
4	5.44209
3	5.53826
2	5.59606
1	5.61534

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